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STAFF PAPER 5

INDEX NUMBERS AND THE SEASONALITY OF QUANTITIES AND PRICES

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I. THE PROBLEM: ITS SETTING AND ITS HARD CORE

1. INTRODUCTION

Each of the major U.S. price indexes covers many commodities that are subject to substantial seasonal fluctuations in quantities consumed or sold.[1] These seasonal changes in quantities are often associated with seasonal fluctuations in prices although they are not the only important source of price seasonalities. The intrayear variation in consumption presents a vexing problem in the construction of indexes designed to measure consistently the movement of prices from month to month and from year to year.

Seasonal changes in quantities and prices may be due to conditions of supply, for example, the short harvest seasons of perishables such as fresh fruits and vegetables or the heavy marketing of cattle and sheep at the end of the grazing season. Or they may be due to conditions of demand, e.g., in the summer the consumption of ice cream is at its peak and that of sweets at its seasonal trough, some meats are considered "heavy" but more is spent on poultry, etc. Much of the variability of food prices in the CPI (and, as a group, foods move faster than any other group within this index) reflects the high seasonality of so many food products. But seasonal influences are also quite substantial among commodities other than foods. In the soft-goods group, apparel is obviously and inevitably subject to such influences, with seasons for new spring and fall lines in clothing being accompanied by higher, and summer and post-Christmas sales by lower, prices. Among the durable goods which include automobiles, furniture, TV, radio, and various household appliances, model changes

[1] We are referring to the Consumer Price Index (CPI) and the Wholesale Price Index (WPI) of the U.S. Bureau of Labor Statistics and to the Indexes of Prices Received and Paid by Farmers of the Agricultural Marketing Service (U.S. Department of Agriculture). Of course, since these indexes measure different things and serve different purposes the implications of the seasonal problem for them are by no means alike. Our concern in some parts of this study will be primarily with the CPI.

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anticipated and introduced in the final months of the year constitute a strong seasonal factor. It is only in the group of services that the seasonal elements are generally weak, in contrast to the above commodity categories.

The timing of the seasonal peaks and troughs, expansions and contractions, varies greatly among the component price series, however, so that these movements offset each other to a large extent, leaving only relatively small seasonal changes in the index as a whole.[2] Nevertheless, seasonal influences may and at certain times did dominate the short-run behavior of a comprehensive measure of average price changes such as the Consumer Price Index. That is, they can temporarily offset or even outweigh the influence of other, primarily cyclical, changes in business conditions. (To be sure, at other times seasonal factors may work in the same direction as cyclical factors, thus reinforcing the effect of the latter.) One illustration is provided by the 1929 developments when the CPI held firm, showing only the normal seasonal changes in foods, while the cyclically sensitive economic activities such as industrial production had already experienced considerable declines.[3]

It would seem that for some purposes, in particular for the analysis of longer-term movements in prices, the solution of the seasonal problem lies in the elimination of seasonal price variation. Techniques for such adjustments are well known and criteria are available according to which the quality of the results can be judged. But even the best seasonal adjustment will not, of course, remedy the shortcomings of the raw (unadjusted) time series to which it is applied. In the present case, the real problem is how to make the index reflect properly the seasonal variation in prices, taking into account the seasonal variation in consumption; it is not how to get the seasonal element out again once it has been adequately measured. In short, the important and difficult issue here is that of proper index measurement, an analytical as well as a practical problem, not the logically subordinate question of technical deseasonalization.

2. SEASONAL WEIGHTS, CHAIN INDEXES, AND THE PROPORTIONALITY CRITERION

To isolate the seasonal problem in price index construction and to simplify the analytical situation, let us assume that the "seasons" can be represented by months of the calendar year and that all change in consumption is seasonal only (no change in annual consumption). Then there would be 12 monthly "market baskets" applicable, respectively, to the Januaries, Februaries, etc., of the successive years. Thus the market baskets would not be constant in the consecutive monthly periods, although they would be constant in the same months or "seasons" of each year. To complete this simplest type of seasonal model, let the price change, too, be of exclusively seasonal nature and such that prices would vary only from month to month but be equal in the same months of each year.

[2] This applies to each of the price indexes reviewed. The overall sensitivity to seasonal factors of the Wholesale Price Index seems to exceed somewhat that of the CPI, while the Indexes of Prices Received and Paid by Farmers (especially the latter) appear to be less subject to such influences. (For a statistical documentation of these statements, see Part III of this paper.)

[3] Cf. Ewan Clague, "The Consumer Price Index in the Business Cycle," *Monthly Labor Review*, LXXXI, No. 6 (June 1958), 616–620. Among the "economic characteristics" of the CPI emphasized by Clague, seasonal factors have indeed a prominent place.

One point, at least, is clear under these hypothetical conditions, namely, that the price index for the current month, P_t, should equal the index for the same month a year ago, P_{t-12}. This follows from the so-called "proportionality" requirement, which underlies one of the tests that Irving Fisher first proposed in *The Purchasing Power of Money* (1911). That an index number of prices should agree with each of the price relatives from which it is derived, if all of these relatives agree with each other, is a criterion that is hardly in need of much explanation or justification.[4] "Proportionality," of course, includes "identity" as a special case. In the present example, identity of prices at $(t-12)$ and t has been postulated, but it is easy to specify somewhat more relaxed assumptions (e.g., admit a steady trend in all prices as well as stable seasonals) such as would result in price level but not in price structure changes from year to year.[5]

Now, to do justice to the seasonal problem, a series of index numbers of price change should reflect as well as possible the seasonal variation in consumption. Since 1887, when Marshall first advanced the chain system and Edgeworth seconded it, many students of index numbers have come to look upon the chain index as the standard statistical solution to changing weights. But careful consideration must be given to the question of how well chain indexes can be applied to the seasonal weight changes with whose specific features they were surely not designed to cope.

It is easy to demonstrate that a chain index with varying weights does not fulfill the test of proportionality (or identity). Table I illustrates this by means of a numerical example in which, for simplicity and space economy, only two commodities and four quarterly seasons are recognized.[6] These hypothetical data embody the assumption that both p' and p'' (that is, "all prices") doubled between periods 0 and 2. Three fixed-base indexes employing different weight systems are shown to satisfy the proportionality test in that each of them has the value of 200 (percent) in period 2 (period 0=100). Of the three corresponding chain indexes with seasonal weights, none passes the test. Again, Table I is based on the assumption that prices and quantities are the same in the same "seasons" (here, quarters) of each year. Thus, on the identity test, the indexes for the same seasons should be equal, too, but they are so only for the fixed-base, not for the chain, formulae.[7]

[4] Fisher regarded this test as "really a definition of an average" (*The Making of Index Numbers*, Cambridge, Mass., 3d ed., 1927, App. I., p. 420). Bortkiewicz pointed out that the requirement is an "obvious consequence" of an even broader concept of a statistical average than that used by Fisher (Ladislaus v. Bortkiewicz, "Zweck und Struktur einer Preisindexzahl," *Nordisk Statistisk Tidskrift*, III, 1924, p. 218; quoted in literal translation).

[5] Then, given the relation $p^k{}_t = \gamma\, p^k{}_{t-12}$ where γ is a constant proportionality factor and p^k is price of any k-th item, the condition to be satisfied by the price index would be $P_t = \gamma P_{t-12}$ (in the exclusively seasonal model introduced before, $\gamma=1$).

[6] This example bears a general resemblance to a short numerical illustration given in Bortkiewicz, op. cit., p. 218, but our model has been developed to emphasize the seasonal aspects which are here of primary interest.

[7] The criterion of proportionality will not be satisfied, except under a special assumption, even by the most sophisticated version of the chain index, the Divisia formula. This approach assumes that prices and quantities change over time by infinitesimal steps, so that the price and quantity indexes can be defined by differential equations and converted into Divisia's "chain indexes" by integration. Divisia's index in its general form can be written as $P^D = dP/P = \Sigma adp/\Sigma qp$. See François Divisia, "L'indice monétaire et la théorie de la monnaie," *Revue d' Economie Politique*, 39 and 40 (1925–26); also separately by Librairie Sirey, Paris. That the Divisia index satisfies the said criterion only if the utility functions are homogeneous was shown by J. Ville, "Sur les conditions d'existence d'une ophélimité totale et d'un indice du niveau des prix," *Annales* de l'Université de Lyon, A 3, 1946 (Engl. translation, 1951, in the *Review of Economic Studies*, Vol. XIX).

TABLE I.—*Selected Measures of Price Change Applied to a Simple Seasonal Model*

Period No.	0	1	2	3	4	5	6
Year	1	1	1	1	2	2	2
Quarter	I	II	III	IV	I	II	III
A. ASSUMED DATA							
Item 1:							
Price (p')	3	5	6	4	Same as for periods 0, 1, 2, . . .[1]		
Quantity (q')	40	20	10	30	Do.[1]		
Item 2:							
Price (p'')	1	4	2	3	Do.[1]		
Quantity (q'')	25	5	15	10	Do.[1]		
B. INDEX NUMBERS*							
(a) Binary comparisons:							
Laspeyres	100.0	206.9					
		100.0	108.3				
			100.0	94.4			
				100.0	66.7		
					100.0	[2] 206.9	([3])
Paasche	100.0	184.6					
		100.0	81.8				
			100.0	75.0			
				100.0	61.7		
					100.0	[2] 184.6	([3])
Fisher ("ideal")	100.0	195.4					
		100.0	94.1				
			100.0	84.2			
				100.0	64.1		
					100.0	[2] 195.4	([3])
(b) Fixed-base indexes (named by corresponding binary formula):							
Laspeyres	100.0	206.9	200.0	162.1	100.0	[2] 206.9	([4])
Paasche	100.0	184.6	200.0	150.0	100.0	[2] 184.6	([4])
Fisher	100.0	195.4	200.0	155.9	100.0	[2] 195.4	([4])
(c) Chain indexes based on formulas by:							
Laspeyres	100.0	206.9	224.1	211.7	141.1	[5] 292.0	[5] 316.3
Paasche	100.0	184.6	151.0	113.3	69.9	[5] 129.0	[5] 105.6
Fisher	100.0	195.4	184.0	154.9	99.3	[5] 194.1	[5] 182.7

[1] That is, we assume that $p'_t=p'_{t+4}$; $q'_t=q'_{t+4}$; $p''_t=p''_{t+4}$; and $q''_t=q''_{t+4}$ (using the subscript t to denote periods as numbered in the first line of the table and listing the variables in the order they appear in the four lines of Section A of the table).

[2] This index for period 5 is equal to the corresponding index for period 1.

[3] The index for period 6 (on base period 5) is equal to the corresponding index for period 2 (on base period 1). The general relation $P_t=P_{t+4}$ holds.

[4] The index for period 6 (on base period 0) is equal to the corresponding index for period 2 (on same base). The general relation $P_t=P_{t+4}$ holds.

[5] These indexes for periods 5 and 6 are not equal to the corresponding indexes for periods 1 and 2, respectively. The relation $P_t=P_{t+4}$ does not hold.

*Formulae used in Section B of the table:

(a) Binary comparisons:

$$\text{Laspeyres } P_{ij}^L=\frac{\Sigma p_j q_i}{\Sigma q_i p_i};$$

$$\text{Paasche } P_{ij}^P=\frac{\Sigma p_j q_j}{\Sigma p_i q_j}$$

$$\text{Fisher ("ideal") } P_{ij}^F=\sqrt{P_{ij}^L \cdot P_{ij}^P}$$

(c) Fixed-base indexes (names indicate correspondence to method of binary comparisons):

$$\text{Laspeyres } P_{0j}^L=\frac{\Sigma p_j q_0}{\Sigma p_0 q_0};$$

$$\text{Paasche } P_{0j}^P=\frac{\Sigma p_j q_j}{\Sigma p_0 q_j}$$

$$\text{Fisher } \bar{P}_{0j}^F=\sqrt{P_{0j}^L \cdot P_{0j}^P}$$

(d) Chain indexes based on formulae by:

$$\text{Laspeyres } \bar{P}_{0t}^L=P_{01}^L \cdot P_{12}^L \cdot P_{23}^L \ldots P_{ij}^L \ldots P_{t-1,t}^L$$

$$\text{Paasche } \bar{P}_{0t}^P=P_{01}^P \cdot P_{12}^P \cdot P_{23}^P \ldots P_{ij}^P \ldots P_{t-1,t}^P$$

$$\text{Fisher } \bar{P}_{0t}^F=P_{01}^F \cdot P_{12}^F \cdot P_{23}^F \ldots P_{ij}^F \ldots P_{t-1,t}^F$$

where $t=0, 1, 2, \ldots$; i=any t; and $j=i+1$.

The major objection to the chain index encountered in the literature is that it will not equal the result of a direct comparison between the first and the last of the periods it covers, except in the trivial case of constant weights. Our criticism of the chain index in the seasonal context does not refer directly to this so-called circular test but is founded on the proportionality criterion. To be sure, the latter when applied to more than two periods can be viewed formally as included in the broader circular criterion, yet the two are certainly not the same. Moreover, the historical controversy about chain indexes and the circular test was primarily concerned with long-term comparisons based on annual data—a very different perspective from our short-run, seasonal view. There is certainly much force in the familiar argument against the circular test and in favor of chain indexes as far as such longtime comparisons are concerned.[8] It is also clear why writers who were thinking in terms of long developments in annual values could and did disregard the proportionality test; economic change over years is complex and relative prices vary continuously, without ever returning to their past constellations. But to ignore the proportionality criterion in dealing explicitly with the seasonal problem would just as surely be wrong, for it is the essence of seasonal movements that they recur from one year to the next in similar patterns which for the most part change only gradually over a number of years.

There are, however, important differences between the various chain formulae with respect to the magnitude and character of the divergencies of these indexes from the values expected under the proportionality test. The Laspeyres chain typically exhibits a marked systematic upward "drift" over time; the Paasche chain, an analogous downward drift. These tendencies are vividly illustrated in Table I. With regard to recurrent seasonal fluctuations, such trends are seriously disturbing.[9] Even when the exaggeration involved in this highly simplified example is heavily discounted, it seems clear that the drifts are too strong for the formulae that produce them to be acceptable.[10] It is true that these drifts are not inherent in the working of the formulae, that is, the latter will produce them under certain,

[8] Briefly restated, the argument is that direct comparisons limited to the price and quantity data for two distant years must contain large errors because they disregard the changes in living standards, habits, etc., that accumulate over time. Binary (year-to-year) comparisons are the most accurate and as the distance in time increases the quality of index measurement deteriorates; by making a chain index out of the annual links, information on prices and quantities in all intervening years is utilized most completely and the inevitable error of the long-distance comparison is minimized. On this view, then, the circular test is not valid theoretically in that it implies the reverse of the above reasoning, namely that the direct comparison P_{0t} (and even the backward direct comparison P_{0t}) is a more accurate measure than the result of a complete, forward-oriented and irreversible as historical time itself, chain of annual links, $\tilde{P}_{0t}$.

[9] There is reason to stress the specific and material nature of the argument behind the above statement. As Ragnar Frisch pointed out, the mere fact of "drifting" does not necessarily imply that the chain method is "wrong" (and the direct index "right"); this issue cannot be resolved by "formal considerations." Cf. R. Frisch, *Econometrica*, Vol. IV, 1936, p. 9.

[10] A few statistical tests and experiments are available, which suggest that the drifts of the chain indexes due to seasonal fluctuations may well be quite pronounced. Erland von Hofsten, *Price Indexes and Quality Changes*, Stockholm, 1952, p. 14, refers to Leo Törnquist. "Finlands banks konsumtionspris index", *Nordisk Tidskrift for Teknisk Økonomi*, København, 1937, as having demonstrated that a week-to-week chain index for food was after 3 years 20 percent higher than a direct comparison. The present author had unfortunately no access to Törnquist's study. Recently, considerable experimentation with seasonally weighted chain indexes and other formulae has been performed at the Bureau of Labor Statistics; its results are summarized in Doris P. Rothwell, "Use of Varying Seasonal Weights in Price Index Construction," *Journal of American Statistical Association*, March 1958, pp. 74–77. Here the 3-year divergence between the Laspeyres chain and direct indexes was similar but slightly larger (close to 28 percent).

not all, circumstances. But it is precisely in the seasonal context that the conditions assuring the occurrence of the drifts will be most often fulfilled (see Section 3b below).

Chain indexes based on some compromise method of crossing formulae or weights will miss the proportionality test much more narrowly, following, as would be expected, an intermediate course between the Laspeyres and the Paasche chains. The Fisher chain in Table I shows some downward drift and other more realistic test calculations also indicate the presence of such slow drifts both in this cross formula and in the Marshall-Edgeworth cross weight chain.[11] But it is likely that under conditions pertinent to the practice of index measurement—a sufficiently large number of component items in the index, less violent period-to-period movements in these data—divergencies such as those yielded by the Fisher chain will not prove seriously disturbing, at least not over a period of a few years at the end of which a revision of the index might be used to "rectify" matters. One must also remember that the stringent seasonality assumptions of the test will not often be closely approximated in practice. After all, seasonal fluctuations are in reality overlaid by trends and cyclical and erratic movements and they are not always well-defined or very regular in themselves.

Thus, on the strength of the charge of "drifting" alone, a strong case can be made against the Laspeyres and Paasche chain formulae, but not against the Fisher chain. The main merit of a chain series, which is that each of the links in the chain uses only those price and quantity changes that belong to the same period and are directly associated with each other, is of course pertinent in the seasonal context as it is in other applications. Hence it is important to ask whether a chain index faces still other difficulties that would tend to offset its admittedly important theoretical advantage.

There is one basic difficulty here that becomes important in connection with seasonal quantity changes, but this difficulty is shared by the chain series with all other conventional price indexes. This concerns the so-called "unique" commodities—items found only in one of the two commodity lists of a binary (two-period) comparison but not in both. Chain indexes of the standard type, like other index numbers computed by averaging price relatives, imply a given list of commodities in two successive pricing periods; that is, they retain in a binary comparison what for fixed-base indexes is true for a number of comparisons (over longer periods of time), namely, that the "market basket" is constant. But the main complication introduced by the seasonal change is precisely that the market basket is different in the consecutive months (seasons), not only in weights but presumably often also in its very composition by commodities. This is a general and complex problem which will have to be dealt with separately at later stages of our analysis.

Finally, turning to the very different matter of practical difficulties associated with the application of the chain method to short-run data with seasonal characteristics, two possibilities must be distinguished.

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[11] A 4 percent downward drift over a period of 10 years (in annual data) was found experimentally for a Fisher chain by Warren M. Persons ("Fisher's Formula for Index Numbers," *The Review of Economic Statistics*, March 1921, p. 110). Recent tests at the Bureau of Labor Statistics reveal a similar drift from year to year in a monthly Marshall-Edgeworth chain with seasonal weights. Cf. Doris P. Rothwell, op. cit., Fig. 77B.

If the seasonal weight patterns are essentially stable from year to year (Table I presents the extreme case where they are constant), then the chain method, which does not take advantage of this stability but rather faces a difficulty in it (the "drift" problem), is of questionable efficiency. If, on the other hand, the intra-annual weight distribution varies considerably over time, then it would seem overzealous to attempt to reflect in the index these numerous short-run changes in weights, many of which are likely to be minor and unsystematic. A monthly or even a quarterly chain index with current weights poses maximum data requirements whose continuous fulfillment can hardly be realistically expected. To try to get reasonably accurate seasonal quantity weights on a current basis would most likely prove an exercise in futility.

3. IMPLICATION OF PRICE-QUANTITY RELATIONSHIPS

a. *Indexes of Price Change and of the Cost of Living.*—The theoretically ideal cost-of-living index may be defined in purely formal terms as the ratio of two money expenditures $V_j = \Sigma p_j q_j$ and $V_i = \Sigma p_i q_i$ which are "equivalent" in the sense that the "typical" consumer in the group to be covered by the index is just as well off at j (spending V_j) as he was at i (spending V_i).[12] Clearly, such an index implies a complete solution to the seasonal problem, as to any other "problem" in cost-of-living measurement. By definition, V_i and V_j are household budget expenditures on equivalent market baskets which will be as similar or as different as required to provide "equal real incomes of utility" (Keynes); this takes care of seasonally motivated as well as any other necessary adjustments in the basket. Given any indicator of equal "well-being," μ or ν (e.g., an indifference function), the index $V_j(\mu,\nu \ldots)/V_i(\mu,\nu \ldots)$ fulfills the proportionality test and the circular criterion in general, identically in μ,ν or any other such indicator.[13]

The theory of cost-of-living measurement acknowledges that the "true" index V_j/V_i is not known. It proceeds from an analysis of the relationship between the two available basic measures of average price change, the Laspeyres and the Paasche indexs (in our notation,

$$\overset{L}{P}_{\dot{i}\dot{j}} \text{ and } \overset{P}{P}_{\dot{i}\dot{j}}$$

respectively), in an effort to establish how these are related to the true cost-of-living indexes.

Assume two "cross combinations" of conditions for our group of consumers: (a) their real income level is still as of period i but they face now a changed structure of prices, that of the next period j; (b) they are confronted with relative prices of period i but their real incomes are those of period j. Let $\bar{q}_i$ denote the quantities that would have been purchased in the first, and $\bar{q}_j$ those that would have

[12] See A. A. Konüs, "The Problem of the True Index of the Cost of Living," *Econometrica*, Vol. 7, No. 1, January 1939, p. 10 (translation of a paper published in Russian in 1924). Definitions which coincide with that given above are also employed in the writings of Gottfried Haberler, *Der Sinn der Indexzahlen*, Tübingen, 1927; A. L. Bowley, "Notes on Index Numbers," *The Economic Journal*, Vol. 38, 1928, pp. 216–237; J. M. Keynes, *A Treatise on Money*, New York, 1930, Vol. I; R. G. D. Allen, "On the Marginal Utility of Money and Its Application," *Economica*, May 1933; Hans Staehle, *International Comparisons of Cost of Living*, International Labour Office, Studies and Reports, series N, No. 20, Geneva, 1934; and Ragner Frisch, op. cit., pp. 10–16.

[13] Frisch, op. cit., p. 13.

been purchased in the second of these hypothetical situations. Then, in accordance with the definition given above, there would be two "true" cost-of-living indexes for the real income levels of i and j, respectively, with formulae much like those of Laspeyres and Paasche except for the crucial substitution of the barred for the simple q's in two instances. These not directly measurable expressions are

$$P^{I}_{ij}=\frac{\Sigma p_j \bar{q}_i}{\Sigma p_i q_i} \text{ and } P^{J}_{ij}=\frac{\Sigma p_j q_j}{\Sigma p_i \bar{q}_j}$$

There are now also two inequalities: [14]

$$P^L > P^I \text{ and } P^J > P^P$$

which are due entirely to changes in the price structure and the response to them of consumers' buying.[15] Implicit in the Laspeyres index is the assumption that demand for any commodity is completely price-inelastic. Because it thus neglects to take into account the adjustments of consumption in favor of items that have become relatively cheaper, the numerator in P^L is too large and P^L exceeds P^I which by definition is free from that error. And again because it implies inelasticity of demand, the denominator in P^P is too large so that P^P is less than P^J which, too, is by definition error-free.

Defining $D_p=(P^L-P^I)+(P^J-P^P)$ and $D_r=P^I-P^J$, we obtain as their algebraic sum the total difference between P^L and P^P, $D_t=D_p+D_r$. If there were no change in real income between periods i and j, D_r would be zero and the difference between P^L and P^P would equal D_p alone, which means that it would be dependent only on the effects of changes in the relative prices and as such be strictly positive. If there is also a change in real income affecting the structure of consumption, then D_r will be non-zero and D_t will depend on the sign and magnitude of D_r as well as on the size of the positive D_p.

A simple yet not ineffective way to evaluate D_r consists in taking a close look at P^I and P^J to compare their relative magnitudes under certain specified conditions. Thus if the group covered by these indexes experiences a net rise in their real incomes between periods i and j, then one would expect that $\Sigma p_j q_j > \Sigma p_i q_i$ but also that $\Sigma p_i \bar{q}_j > \Sigma p_i q_i$. In other words, both the numerator and the denominator of P^J would then be larger than the corresponding components of P^I. If the difference between the numerators were larger than that between the denominators, P^J would exceed, and in the reverse case it would fall short of, P^I. There does not seem to be any reason for either of these eventualities to have a higher probability of occurrence than the other; and the parallelism of the two inequalities works to make the difference between P^I and P^J small. Analogous considerations apply to the case of a net decline in real incomes between periods i and j, where the expected relations are $\Sigma p_j q_j < \Sigma p_i q_i$ and $\Sigma p_i \bar{q}_j < \Sigma p_i q_i$. The inference to be drawn in each case is that the differences, D_r, between the cost-of-living indexes P^I and P^J, or between their fixed-base equivalents, are likely (a) to bear signs that do not vary systematically over time, and (b) to be small and, on the aver-

[14] For simplicity, the subscripts of the indexes are henceforth omitted.

[15] Cf. Melville J. Ulmer, *The Economic Theory of Cost of Living Index Numbers*, New York, 1949.

age, zero. Hence the total difference $D_t = P^L - P^P = D_p + D_i$ would tend to have the general order of magnitude and the sign (+) of D_p. These conclusions are consistent with the available evidence.[16]

Changes in real incomes are primarily a cyclical and a trend phenomenon, and presumably of relatively little importance in the shorter run. In the seasonal context, in particular, changes of relative prices and of quantities consumed can be expected to dominate the scene. Table I shows this in a highly exaggerated form[17] but without falsifying the direction in which these factors work on most (although by no means on all) occasions.[18] The model assumes a negative correlation between the price and quantity relatives. It yields Laspeyres indexes consistently exceeding the corresponding Paasche indexes (see the section "binary comparisons" in Table I).

b. *Correlation and Dispersion of Price and Quantity Relatives.*—Another instructive approach to the analysis of the relation between P^L and P^P has been developed by Bortkiewicz and applied in empirical work on international cost-of-living comparisons by Staehle. For convenient notation, define

$$x = \frac{p_j}{p_i},\ y = \frac{q_j}{q_i},\ \text{and } w = p_i q_i$$

Then we can write (omitting the subscript ij in the index symbols)

$$P^L = \frac{\Sigma wx}{\Sigma w};\ P^P = \frac{\Sigma wxy}{\Sigma wy};\ \text{and } Q^L = \frac{\Sigma wy}{\Sigma w}$$

where the last expression is a *quantity* index (Laspeyres). By their definitions, the weighted coefficient of correlation between x and y (r_{xy}) and weighted variances of these variables (σ^2_x and σ^2_y) are

$$r_{xy} = \frac{\Sigma w(x - P^L)(y - Q^L)}{\sigma_x \sigma_y \Sigma w};\ \sigma_x^2 = \frac{\Sigma w(x - P^L)^2}{w}$$

$$\text{and } \sigma_y^2 = \frac{\Sigma w(y - Q^L)^2}{\Sigma w}$$

The following equation can be shown to hold:[19]

$$-\frac{D_t}{P^L} = \frac{P^P - P^L}{P^L} = r_{xy} \cdot \frac{\sigma_x}{P^L} \cdot \frac{\sigma_y}{Q^L}$$

[16] See M. J. Ulmer, op. cit., pp. 55–58, where some annual retail price data for 1929–40 are shown to yield very low positive values of D_t. (They are based on a fixed-weight Laspeyres and a variable-weighted Paasche index, average 0.3 percent of either of these measures, and seem to show a slight positive cyclical pattern.)

[17] This has two reasons: (1) The assumed fluctuations in prices and quantities are very large, as are the implied movements in the relative prices and expenditure weights, and (2) there are only two items in the example. Cf. Fisher, op. cit., app. II, "The greater the number of commodities in an index number of prices, the less is the index number affected by a change in weights, or in price relatives" (p. 450). About the empirical effects of different scales of coverage and types of weighting, see also Wesley C. Mitchell, *The Making and Using of Index Numbers*, U.S. Bureau of Labor Statistics, pt. I, of Bulletin 173, 1915, reprinted as Bulletin 656, 1938 (secs. IV5 and 156).

[18] See Section 3d below.

[19] Derived by Bortkiewicz, op. cit., pp. 13–14. An earlier version of this analysis is given in Bortkiewicz's first article in *Nordisk Statistisk Tidskrift*, II, 1922, pp. 374–379.

Thus the divergence between P^L and P^P, standardized in terms of P^L, is found to depend on three factors: (1) the coefficient of correlation between the price and quantity relatives, p_j/p_i, and q_j/q_i, and (2, 3) the coefficients of variation of these relatives (each of these coefficients being weighted by means of $w=p_iq_i$).

The ratio σ_y/Q^L applied to two seasons i and j would measure the extent to which the structure of consumption differs between these periods for households with specified characteristics. The ratio σ_x/P^L would similarly measure the divergence between the i-th and the j-th relative price systems. Either ratio could theoretically be zero (if q_j were proportional to q_i, or p_j to p_i, for all commodities, i.e., if $q_j/q_i=$const. or $p_j/p_i=$const.). Actually, either can be expected to be positive, of course, but most likely less than one. The distribution of consumption in periods i and j would have to be very asymmetrical—associated with a very large dispersion of the quantity relatives y—in order for σ_y to reach values exceeding Q^L. The case of $\sigma_x/P^L>1$ is still less probable: that ratio would more likely than not be smaller than σ_y/Q^L, although the two may not be widely different.

Since both σ_x/P^L and σ_y/Q^L are positive, the sign of the total difference $D_t(=P^L \quad P^P)$ must be opposite to the sign of r_{xy}. Thus in the case of a negative correlation between price and quantity relatives, which is the assumption we have been making so far, D_t will be positive. The analysis also suggests that D_t/P^L, when based on a large number of common consumption items, should not be large: its value is the *product* of three factors each of which is a proper fraction. Still, its value might be quite respectable as shown by the following, perhaps not implausible, example: assuming r_{xy}, σ_x/P^L, and σ_y/Q^L are, respectively, -0.6, 0.3, and 0.4, the resulting D_t/P^L would be 0.072 or somewhat more than 7 percent.

A similar analysis may be used to explain the relation between chain comparisons and the corresponding direct comparisons, say $\tilde{P}^L{}_{ot}$ and $P^{L}{}_{ot}$. Restricting the chain to a single link of two indexes without loss of generality and defining

$$x_k=\frac{p_k}{p_j},\ y_j=\frac{q_j}{q_i},\ \text{and } w_j=q_ip_j$$

(i, j, and k denoting three successive periods), we have

$$\frac{P^L_{ij}\cdot P^L_{jk}}{P^L_{ik}}=1+\frac{r'_{xy}\sigma'_x\sigma'_y}{\bar{x}_k\ {}_j}$$

Here r'_{xy} is the coefficient of correlation between x_k and y_j, $'_x$ and σ'_y are the respective standard deviations of these variables, and $\bar{x}_k$ and $\bar{y}_j$ are their means, all of these expressions being weighted with w_j.[20] It is evident that if r'_{xy} is positive, the left-hand expression will

[20] The above equation seems to convey the analytical situation and its implications somewhat more directly than the original version given in Bortkiewicz, *Nordisk Statistisk Tidskrift,* III, p. 211. In the present notation, the Bortkiewicz relations reads

$$P^L{}_{ij}P^L{}_{jk}-P^L{}_{ik}=\frac{r'_{xy}\sigma'_x\sigma'_yP^L{}_{ij}}{Q^P{}_{ij}}$$

That the two versions are equivalent is easily verified, once it is realized that

$$\bar{x}_k=P^L{}_{ik}/P^L{}_{ij} \text{ and } \bar{y}_j=Q^P{}_{ij}$$

be larger than one, i.e., the chain Laspeyres $(P^L_{ij} \cdot P^L_{jk})$ will give a higher result than the direct Laspeyres (P^L_{ik}). By the same token, a negative r'_{xy} would make $P^L_{ik} > P^L_{ij} \cdot P^L_{ik}$.

Now the former of these two eventualities has on occasion been presented as an unqualified rule.[21] Actually, a sweeping generalization to this effect cannot be made, since the outcome will depend on the conditions of the case, for example on the length of the unit period of the comparisons.[22] However, as far as short-run seasonal elements of price and quantity movements are concerned, there are some good reasons, as well as empirical evidence, to expect that $P^L_{ik} < P^L_{ij} P^L_{jk}$ would indeed prove to be the dominant tendency in practice. Two points must be made: (1) We can assume that taking "the season" as a unit period, the correlation between price and quantity relatives on a simultaneous basis is likely to be negative for a large number of products. (2) Seasonal variations may be conceived as deviations from an annual average, so that they imply a "normal": rises above and falls below that level will tend to succeed each other in compensatory sequences over the year for both the price and the quantity relatives. Now the combination of (1) and (2) makes it probable that when these relatives are taken with a *lag*, which is the case here where we consider q_j/q_i and P_k/P_j, their correlation, as measured by r'_{xy}, would be positive. This is the situation represented in Table I which implies an association between the price relatives and the quantity relatives that meets the above conditions. It is because of this that the resulting index numbers show the familiar "drifts."

c. *Unique Commodities.*—Can the analysis of the previous section help us in dealing with the problem of "unique" commodities? It has been observed that the sequence of seasons produces substantial changes not only in the amounts of the same goods purchased at different times of the year but often also in the variety of the goods purchased. For many items the supply (or demand) is heavily concentrated in certain seasons; for some items it is entirely confined to this or that part of the year. Is it possible, e.g., to have the expression σ_y/Q^L cover two sets of commodities that include some items encountered only in one but not the other of the compared periods? And, if so, what might be learned from such a measure?

For any item that appears in the i-th but not in the j-th basket, the quantity relative q_j/q_i is zero. The Laspeyres quantity index Q^L can be computed for a situation in which some of the q_j are zero, either as a weighted average of quantity relatives, $\Sigma(q_j/q_i)\,q_i p_i/\Sigma q_i p_i$, or as a ratio of aggregates, $\Sigma q_j p_i/\Sigma q_i p_i$. The two forms are here equivalent, just as they are in the normal case of index-making practice where only positive (reported or estimated) q_j are used.

For any item that appears in the j-th but not in the i-th basket, the quantity relative q_j/q_i, and consequently Q^L as a weighted average of such relatives, cannot be computed. Where q_i is zero there is no corresponding market price p_i, so that the aggregative form $\Sigma q_j p_j/\ \Sigma q_i p_i$ cannot be extended beyond the intersection of the two sets of commodities either (unless hypothetical instead of actual market prices are

[21] See Ragnar Frisch, "Annual Survey of General Economic Theory: The Problem of Index Numbers," *Econometrica*, Vol. IV, 1936. p. 9.

[22] This dependence was noted, without further elaboration, by Bortkiewicz, op. cit., p. 219.

used for p_i). But a Paasche quantity index can be obtained from the formula $Q^P = \Sigma q_j p_j / (q_i/q_j) q_j p_j$, where some of the relatives q_i/q_j are now zero; and the analysis of the difference D_t can be worked out in terms of the Paasche as well as the Laspeyres indexes.[23]

The weighted relative variance of the quantity ratios is

$$\sigma_y^2 = \frac{\Sigma w(y - Q^L)^2}{\Sigma w} = \frac{\Sigma w.y^2}{\Sigma w} - (Q^L)^2,$$

where $y = q_j/q_i$ and $w = q_i p_i$. The case of $q_j = 0$ is here again very simple. Such an item contributes a zero y to $Q^L = \frac{\Sigma wy}{\Sigma w}$ in the second part of the above expression and similarly a zero y^2 to the first part of it. For $q_i = 0$, the "Laspeyres-type" variance σ_y^2 cannot be computed but the "Paasche-type" variance $\sigma_{y'}^2$ (see footnote 23) can. The latter can be written as $\Sigma w'(y')^2/\Sigma w' - (1/Q^P)^2$, where $Q^P = \Sigma w'/\Sigma w'y'$. For each item with $q_i = 0$ ($q_j > 0$), $y' = q_i/q_j$ will equal zero.

The situation with respect to price relatives and price indexes is different. In our first case ($q_i > 0$; $q_j = 0$), the price of the commodity is positive at i, nonexistent at j. It is not possible simply to parallel the treatment on the quantity side and include the price relative p_j/p_i for this item at the value zero in the computation of the Laspeyres index P^L. To do so would clearly involve a logical error (absence of a market price is not identical with the existence of a zero price) as well as a distorted measurement of the average price change (disappearance of an item from the market does not per se lower the index and should not be permitted to have this effect). The same consideration applies *mutatis mutandis* to the case of $p_j > 0$ and p_i nonexistent ($q_i = 0$). It is valid for the Paasche as well as for the Laspeyres price index. There is simply no escape from the truism that any comparison of two magnitudes such as p_i and p_j requires that both of them be actually given. If either is not directly observable, then, under the method of item-by-item comparisons, it must be estimated or else the item concerned must be omitted from the index altogether, and not just from that part of the index relating to the period for which p is not available. Being true generally for the price relatives and their aver-

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[23] Define $x' = \frac{p_i}{p_j}$, $y' = \frac{q_i}{q_j}$, and $w' = p_j q_j$. Then

$$\frac{1}{P^P} = \frac{\Sigma w'x'}{\Sigma w'};\ \frac{1}{Q^P} = \frac{\Sigma w'y'}{\Sigma w};\ \text{and}\ \frac{1}{P^L} = \frac{\Sigma w'x'y'}{\Sigma w'y'}$$

$$\text{We get } \sigma_{x'}^2 = \frac{\Sigma w'\left(x' - \frac{1}{P^P}\right)^2}{\Sigma w'};\ \sigma_{y'}^2 = \frac{\Sigma w'\left(y' - \frac{1}{Q^P}\right)^2}{\Sigma w'};\ \text{and}$$

$$r_{x'y'} = \frac{\Sigma w'\left(x' - \frac{1}{P^P}\right)\left(y' - \frac{1}{Q^P}\right)}{\sigma_{x'}\sigma_{y'}\Sigma w'}$$

The equation for the relative D_t has now the form:

$$\frac{-D_t}{P^L} = \frac{\frac{1}{P^L} - \frac{1}{P^P}}{\frac{1}{P^P}} = \frac{P^P - P^L}{P^L} = r_{x'y'} \cdot \sigma_{x'}\sigma_{y'} \cdot P^P \cdot Q^P.$$

ages, the conventional price indexes, this argument is of course also applicable to variances of price relatives such as $\sigma_x^2 = \frac{\Sigma w(x-P^L)^2}{\Sigma w}$ (where $x=p_j/p_i$ and $w=p_iq_i$). This expression, too, cannot be extended to cover heterogeneous aggregates, i.e., different through overlapping sets of commodities, except through the use of some hypothetical prices.

In view of the above, one must conclude that the Bortkiewicz analysis of D_t cannot as a whole be consistently applied to commodity sets that include unique goods. This imposes a considerable limitation upon its value for the treatment of the seasonal problem. Of those parts of the analysis that retain interest for the case of unique commodities, the ratio σ_y/Q^L is the most important. This expression can be regarded as a measure of the difference in the structure of consumption between the two situations or periods computed. It may be written as

$$\frac{1}{Q^L}\sqrt{\frac{\Sigma w(y-Q^L)^2}{\Sigma w}} = \sqrt{\left(\frac{1}{Q^L}\right)^2\left[\frac{\Sigma wy^2}{\Sigma w}-(Q^L)^2\right]} = \sqrt{\frac{\Sigma q_ip_i\Sigma q_jp_i\left(\frac{q_j}{q_i}\right)}{(\Sigma q_jp_i)^2}-1}.$$ [24]

The value of this Laspeyres-weighted coefficient of variation is obviously an increasing function of the dispersion of the quantity relatives q_j/q_i from their weighted average Q^L. But it is thus implicitly also an increasing function of the importance of those commodities that appear in the i-th but not in the j-th "market basket." With each replacement of a positive by a zero q_j, Q^L is lowered and σ_x raised. On both counts, then, the value of the relative variance or standard deviation of the quantity relatives is increased. The accompanying tabulation provides a simple hypothetical example.[25]

Data							Results		
Variable	Items					Model	σ_x	Q^L	$\frac{\sigma_x}{Q^L}$
	1	2	3	4	5				
p_i	3	2	1	5	4				
q_i	4	5	6	2	1	I	1.014	1.119	0.906
q_j	2	3	4	3	4				
q_j	0	3	4	3	4	II [1]	1.118	0.976	1.145
q_j	0	0	4	3	4	III [1]	1.195	0.833	1.434

[1] p_i and q_i as in model I.

Where instances of $q_i=0$ ($q_j>0$ occur,

$$\sigma_y^2 = \frac{\Sigma w'(y')^2}{\Sigma w'} - \left(\frac{1}{Q^P}\right)^2$$

is the expression to be evaluated (see footnote 23). This is the weighted relative variance of the quantity relatives q_i/q_j. The ratio

[24] For the definitions of the symbols used, see Part I, Sections 3b and 3c.

[25] Its results are, of course, again greatly exaggerated, for reasons analogous to those noted before in connection with Table I (see Part I, Section 2a and footnote 17). In realistic cases σ_x/Q^L would be expected to be much lower than one. The presence of unique commodities would indeed work strongly to raise the value of that ratio, but there will be relatively few such items in representative market baskets.

corresponding to σ_y/Q^L in the analysis of the Laspeyres terms is here $\sigma_{y'}/(1Q^P)$. Where cases of $q_j=0$ occur along with those of $q_i=0$, both the Laspeyres- and the Paasche-type measures would be needed. The analysis, then, would consist of two parts, and for an appraisal of the total change the results of the two should be combined. It should also be instructive to have the above expressions computed in two variants, one inclusive and the other exclusive of the unique commodities. This would permit separate estimation of the influence of the factors of dispersion and nonhomogeneity.

The analysis of differential consumption structures has been put to some interesting empirical uses with little concern for the difficulties discussed in these pages. Minimization of the difference between the structures of consumption in two different price situations has been proposed as a method of ascertaining "equivalent" income levels whose ratio approximates the theoretical cost-of-living index.[26] Let a series of "incomes" in the base situation be distinguished—values of $\Sigma q_i p_i$ for various q_i, baskets, and the corresponding prices—and let each such value be compared with a series of incomes for the j-th situation, or different combinations for $\Sigma q_j p_j$. For each pair of these aggregates, a value of σ_z/Q^L (or Δ, see footnote 26) can be calculated. Empirically, a tendency was found for each of such series of comparisons to yield a fairly well-defined minimum value for these measures of dissimilarity of the quantities consumed. The lowest of the minima were used by Staehle in his international comparisons as means of selecting pairs of incomes regarded as most nearly equivalent in terms of living standards. Staehle's results were found encouraging, although further studies are needed to reach firmer conclusions on the usefulness of his method. The possibility of applying the latter in an approach to the task of constructing an index with seasonal weights is contemplated later in this paper (see Part II, Section 9 below).

The existence of commodities that are marketed only at certain times of the year (the "unique goods" in the seasonal context) dramatizes the index number problem posed by the seasonality of quantities sold. No conventional price index formula can handle a situation in which the "market basket" varies between two consecutive periods. This is the hard core of the seasonality problem. To make real sense economically, the solution of this problem must seek an approximation to constant-utility indexes through the use of seasonal goods complexes that approach equivalence in the eyes of the representative consumer or producer.

d. *Seasonal Shifts in Demand and Seasonal Indifference Curves.*[27]—Much of the preceding discussion was related to seasonalities whose

[26] Hans Staehle, op. cit.; see also articles by the same author in *Archiv für Sozialwissenschaft,* June 1932, *Econometrica,* January 1934, and *The Review of Economic Studies,* June 1935. Staehle uses as his measure of "dissimilarity" between the quantity complexes q_i and q_j the expression

$$\Delta = \frac{\Sigma w |y - Q^L|}{\Sigma w} \cdot \frac{1}{Q^L}$$

He notes that Δ bears a close family relationship to the Bortkiewicz measure

$$\sqrt{\frac{\Sigma w (y - Q^L)^2}{\Sigma w}} \cdot \frac{1}{Q^L}.$$

Staehle's Δ can vary between 0 and 2 (and Frisch, op. cit., p. 30, observes that it will equal 2 only when none of the q_j goods occur in q_i). The values of Δ for our models I, II, and III are 0.652, 0.846, and 1.101, respectively.

[27] The author is indebted to Professor Martin Bailey (University of Chicago) for helpful criticism and suggestions relating to this section.

source lies on the supply side. This category is indeed particularly important in practice. Thus, production of many foods undergoes pronounced intra-annual fluctuations. The relation between the monthly price and quantity ratios is negative because changes in supply cause movements along essentially stable demand functions.

In some cases, however, seasonal variation is due primarily to demand rather than supply changes. For example, the demand for gasoline increases considerably during the summer when cars are used more extensively, but crude petroleum is produced and refined continuously throughout the year with little seasonal change. In this category there is no reason for an inverse association between quantities and prices over the course of the seasons, but instead there are the possibilities of (a) positive correlation or (b) no correlation. If production does increase at the time of the seasonal rise in demand and if this is accompanied by rising marginal costs, then the price can be expected to go up in the demand season. This, then, is the positive correlation case (a). But if the supply curves for the given product(s) are highly elastic over the pertinent range of demand variation or if the peak seasonal demand is met at no substantial additional or specific costs from stock of output produced in, and carried over from, the low-demand season, then the price need not increase at all at the time when sales do. In these situations, the price-quantity correlation over the seasons would be zero or close to it (b).

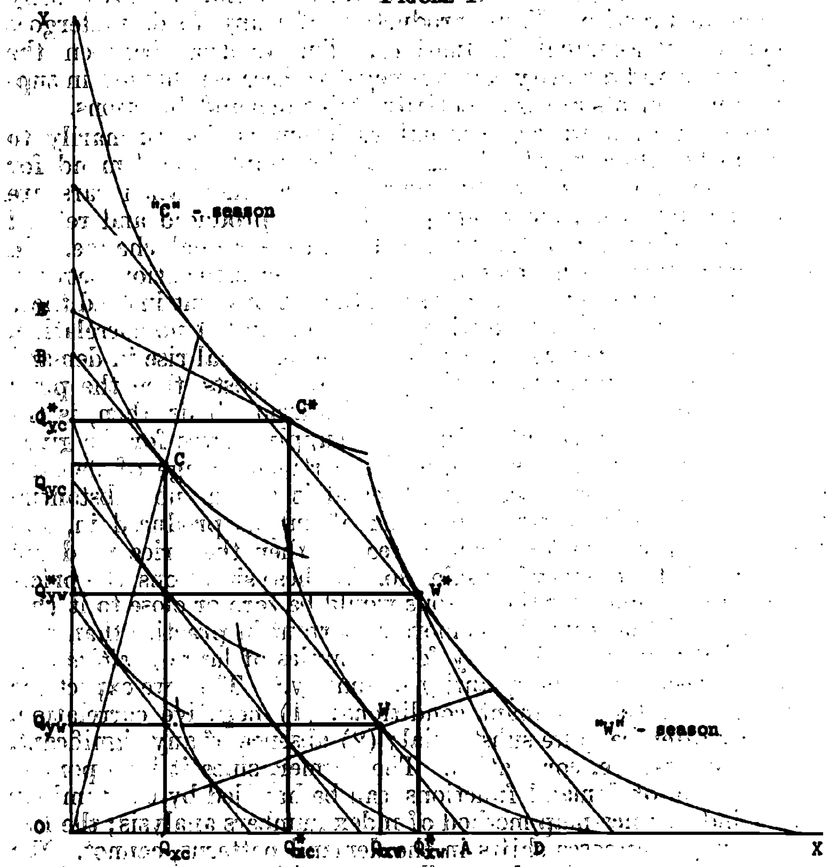
It is believed that case (b) is more important in practice than case (a), i.e., that there are relatively few examples of large *positive* correlation between price and quantity seasonally.[28] Thus we expect the following to be the dominant conditions: (1) negative correlation, which may often be quite substantial; (2) absence of any significant, or some low positive, correlation. The former, since it incorporates a stable system of demand functions, can be handled by the familiar constant-indifference-map method of index numbers analysis; the latter, since it presupposes shifts in the demand patterns, cannot. We submit that one logical and plausible way of looking at some situations that are here involved is to assume a seasonal rotation of indifference curves or the existence of different sets of such curves characteristic of the different seasons.

Figure 1 refers to a simple two-goods, two-seasons case. Suppose the year is about evenly divided into a "warm" and a "cold" season, *W* and *C*. Let *X* be an article used primarily in the *W* season, e.g., a light suit, and *Y* an article used primarily in the *C* season, e.g., a heavier suit, both items being sufficiently well defined and measurable in some standardized units.

There are now two sets of indifference curves, one for the *C* and one for the *W* season. The *C* curves start from the *Y* axis and decline markedly at first but then flatten off sharply, indicating that a sufficiently large quantity of the commodity *Y* can replace *X* entirely in this season and that some quantity of *Y* will be purchased in any

[28] In some instances, the existence of positive correlation appears to make little sense, but even there, of course, there is no point in ignoring its possibility. Thus discounts may be offered in off-season months on goods providing seasonal consumption services, e.g., on air-conditioners in the winter, and yet, despite the durability of the product, only a small proportion of the annual air-conditioner sales may be made during the cool-weather part of the year. In other instances, there may be more logical justification for a positive price-quantity correlation, as when some significant storage or inconvenience costs are incurred by the off-season buyer (e.g., coal purchases for domestic heating purposes in the spring) or a restriction exists on choice (e.g., swimming suit purchases in autumn).

FIGURE 1



events—no increase in X can balance off a decrease in Y below that amount, so that the latter represents a minimum seasonal quantity demanded of Y. Thus it is only within a certain range of the Y-quantities that X can be substituted for Y as shown by the indifference patterns. The same applies *mutatis mutandis* to the W-set of the curves. These, of course, start from the X-axis and the "flattening" takes here the form of a gradual approach to verticality. The rôles of the two items are reversed: there is a minimum for X below which no substitution of Y for it is possible.

Let us suppose that the ratio of the price of X to the price of Y is the same in the two seasons despite the seasonality of demand; production along a horizontal segment of the marginal cost curve in each of the firms making X or Y throughout the year would exemplify the possibility of such a situation. Thus the slope of the budget line, such as the line AB in Figure 1, is given and constant. The set of indifference curves representative of the W-season is so placed in the consumer's preference field that the equilibrium (tangency) solution consists in a combination of a large quantity purchased of X and a small quantity purchased of Y (compare $0Q_{xw}$ and $0Q_{yw}$ in Figure 1).

Analogously, the position of the C-season indifference curves is such that, even with the relative prices of X and Y unchanged, little is bought of the former and much of the latter product (cf. $0Q_{xc}$ and $0Q_{yc}$).

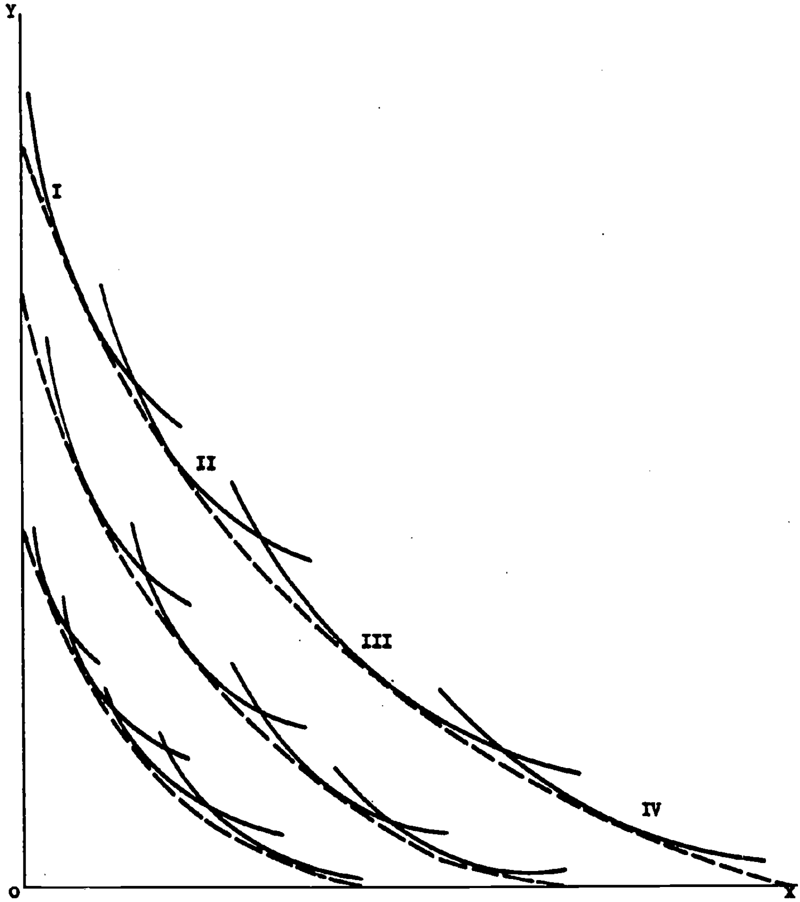
The other possibility is that of seasonal shifts in relative prices in favor of the item experiencing the slack, off-season demand. Thus let the price of X relative to the price of Y be lower in the C than in the W season. The slope of the budget line, which is equal to the price ratio px/py, would then be less in the former in the latter season, as illustrated by the lines EC^* and DW^* in Figure 1. Compared to the constant price-ratio case (applied to the same pair of seasonal indifference curves), the normal result here will be, of course, an increased quantity demanded of the off-season item and a decreased quantity demanded of the in-season item. Comparisons in time between the seasons will show a positive quantity-price correlation: more X is demanded at a higher price in W than in C (and conversely more Y is demanded at a higher price in C than in W).

In the case of a few sharply distinguished seasons, as in our two-season model, annual indifference curves are apt to be mere average constructs with little, if any, analytical significance. But if the interseasonal shifts are more frequent and continuous, the seasonal patterns may be conceived as superimposed upon a conventional indifference map representative of the consumer's preference system in longer time periods. Such a situation is illustrated in Figure 2 where the longer-term indifference curves are shown in broken, the seasonal curves in solid, lines.

For comparability with the latter, the former curves are reduced in scale, from "per annum" to "per season" units. Four seasons are distinguished explicitly, but a similar picture for a larger number of seasons with still less interseasonal discontinuity can be easily imagined. The diagram simply follows the notion, which ought to be often true, that the possibilities for substitution will be greater in the longer time periods than in the very short run.

In Figures 1 and 2 we have assumed that X and Y are good substitues over broad quantity ranges in each season. But in some cases the substitutability range may be very narrow, e.g., as short as AB or CD in Figure 3. In the extreme event of zero substitutability, X only would be demanded in the W season and Y only in the C season. The map would then consist of straight lines rising upward from, and perpendicular to, the X-axis and running to the right from, and perpendicular to, the Y-axis (such as AA, $A'A'$ . . . and CC, $C'C'$ . . . in Figure 3). Viewed from their intersection points upward and rightward, these lines form a set of angular "indifference curves" such as are known from the analysis of the relationship of perfect complementarity (see AIC, $A'I'C'$, etc., in Figure 3). But here again caution is needed lest one concentrate on annual patterns that may be spurious or misleading at the expense of seasonal patterns that have real significance. Thus whether the seasonal components of the "map" are of the initially curved sort (BIA, $B'I'A'$ . . . and DIC, $D'I'C'$ . . .) or straight lines throughout (AIA, $A'I'A'$ . . . and CIC, $C'I'C'$ . . .), the same angular patterns—AIC, etc.—are obtained in either case in the annual, two-

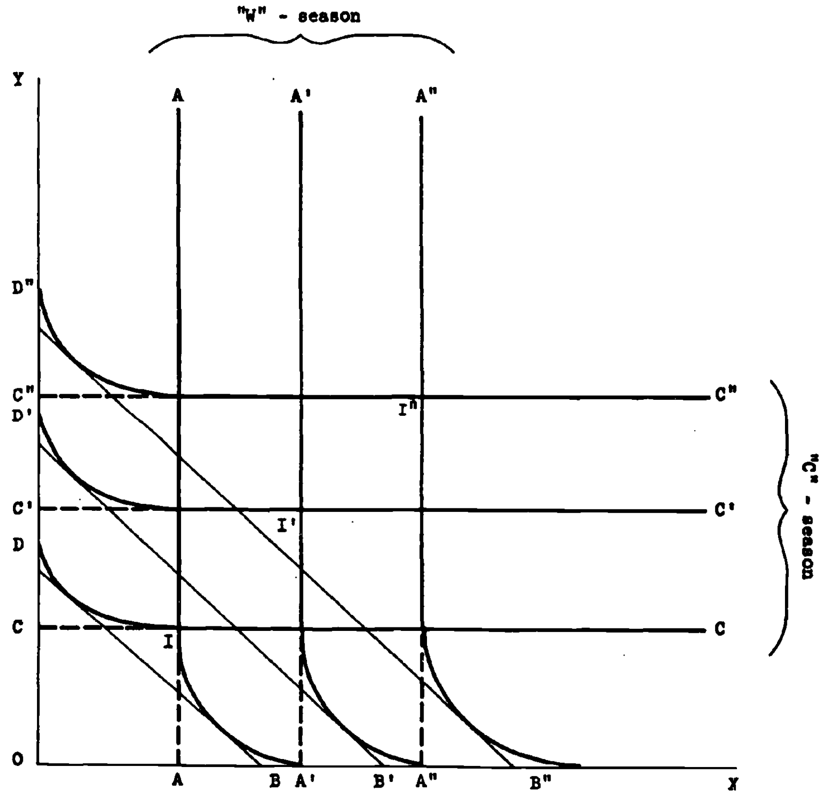
FIGURE 2



season view. But obviously the two cases are in fact quite different: the former allows some substitutions similar to those shown in Figure 1 (see the budget lines in Figure 3), while the latter allows no substitutions whatsoever.

A comment may be added here on the interpretation of the seasonal indifference maps. They formalize different patterns of consumer preferences but this type of variation does not indicate any basic changes in tastes and habits. The taste systems of individuals and families reflect, among other things, the established seasonal patterns of living, but they are not appreciably or systematically altered by the short-run and periodic changes in the natural or social environment that constitute the complex "seasonal factor." The role of the latter is thus seen as static rather than dynamic. Seasonal variation in quantities consumed merely represents a periodic variation in the means whereby people satisfy the same wants over the course of the year. The ends themselves can be viewed as definite, known, and seasonally invariant. Basically, the seasonal problem does not in-

FIGURE 3



volve either really "new" products or "new" wants. In short, the concept of the "taste system" includes a provision for the regular seasonal variation; the system itself is not regarded as changing from one season to another. (Tastes do change over time, of course, but as a rule gradually, under the influence of more enduring, long-term factors.)

Turning from analysis to index measurement, it must be noted that the model in which only quantities undergo seasonal change presents no problems for the conventional price index formulae. Table II illustrates a case where the relative valuation of the component items remain unchanged over the seasons while the quantity relations vary, so that prices and quantities are not correlated in their interseasonal movements. As shown in this example, the formulae of Laspeyres and Paasche (and thus also their "crossing," as represented by the Fisher index) give identical results here. Moreover, chaining the binary comparisons results in this case in the same time series as that obtained by the fixed-base approach. The series satisfies, among others, the proportionality test. The two methods, then, are equivalent here and either is, in terms of the traditional criteria, satisfactory.

TABLE II.—*Selected Price Index Measures for a Model with Seasonal Change in Quantities and Stable Relative Prices*

Period no.	0	1	2	3	4	5	6
Year and quarter (in parentheses)	1(I)	1(II)	1(III)	1(IV)	2(I)	2(II)	2(III)
A. ASSUMED DATA							
Item 1:							
Price (p')	3	3.6	4.8	6	Same as for periods 0, 1, 2...[1]		
Quantity (q')	40	10	10	40	Do.[1]		
Item 2:							
Price (p'')	1	1.2	1.6	2	Do.[1]		
Quantity (q'')	25	50	50	25	Do.[1]		
B. INDEX NUMBERS[2]							
(a) Binary comparisons (Laspeyres, Paasche, Fisher).[3]	100.0	120.0					
		100.0	133.3				
			100.0	125.0			
				100.0	50.0		
					100.0	[4] 120.0	([5])
(b) Fixed-base indexes[3]	100.0	120.0	160.0	200.0	100.0	[4] 120.0	([5])
(c) Chain indexes[3]	100.0	120.0	160.0	200.0	100.0	[4] 120.0	([5])

[1] That is, we assume, that $p'_t = p'_{t+4}$; $q'_t = q'_{t+4}$; $p''_t = p''_{t+4}$; and $q''_t = q''_{t+4}$ (using the subscript t to denote periods as numbered in the first line of the table).

[2] The formulae used in this table are the same as those used in Table I. See footnotes to Table I for identification of the formulae.

[3] The formulae of Laspeyres, Paasche, and Fisher all give the same results for this class of indexes.

[4] This index for period 5 is equal to the corresponding index for period 1.

[5] The index for period 6 is equal to the corresponding index for period 2. The general relation $P_t = P_{t+4}$ holds.

On the other hand, difficulties of the same kind as those encountered in dealing with the model in which prices and quantities are negatively correlated (the basic situation illustrated in Table I) also attach to the case of positive correlation between prices and quantities, as exemplified in Table III. In both models systematic differences obtain between the results of the formulae of Laspeyres and Paasche, but the indexes reverse their roles in the two situations. In the familiar case of Table I, the Laspeyres indexes exceed the corresponding Paasche indexes throughout, and the Laspeyres chain has an upward, and the Paasche chain a downward, drift. In Table III, the Paasche indexes are larger than the Laspeyres indexes, and it is the Paasche chain that shows an upward, and the Laspeyres chain that shows a downward, drift.

TABLE III.—*Selected Price Index Measures for a Model in Which Prices and Quantities Are Positively Correlated Over the Seasons*

Period no.	0	1	2	3	4	5	6
Year and quarter (in parentheses)	1(I)	1(II)	1(III)	1(IV)	2(I)	2(II)	2(III)
A. ASSUMED DATA							
Item 1:							
Price (p')	5	2	3	5	Same as for periods 0, 1, 2 . . .[1]		
Quantity (q')	40	10	20	40	Do.[1]		
Item 2:							
Price (p'')	4	8	6	4	Do.[1]		
Quantity (q'')	25	50	35	25	Do.[1]		
B. INDEX NUMBERS[2]							
(a) Binary comparisons:							
Laspeyres	100.0	93.3					
		100.0	78.6				
			100.0	88.9			
				100.0	100.0	(3)	
Paasche	100.0	168.0					
		100.0	84.4				
			100.0	111.1			
				100.0	100.0	(3)	
Fisher ("ideal")	100.0	125.2					
		100.0	81.4				
			100.0	99.4			
				100.0	100.0	(3)	
(b) Fixed-base indexes:							
"Laspeyres"	100.0	93.3	90.0	100.0	100.0	(3)	
"Paasche"	100.0	168.0	112.5	100.0	100.0	(3)	
"Fisher"	100.0	125.0	100.6	100.0	100.0	(3)	
(c) Chain indexes:							
Laspeyres	100.0	93.3	73.3	65.2	65.2	[4] 60.8	
Paasche	100.0	168.0	141.8	157.5	157.5	[4] 264.6	
Fisher	100.0	125.2	102.0	101.3	101.3	[4] 126.9	

[1] That is, we assume that $p'_t=p'_{t+4}$; $q'_t=q'_{t+4}$; $p''_t=p''_{t+4}$; and $q''_t=q''_{t+4}$.
[2] The formulae used in this table are the same as those used and identified in Table I.
[3] The index for period 5 is equal to the corresponding index for period 1. The general relation $P_t=P_{t+4}$ holds.
[4] These indexes for period 5 are not equal to the corresponding indexes for period 1. The relation $P_t=P_{t+4}$ does not hold.

4. AN OVERALL VIEW OF THE PROBLEM AND THE NEXT STEPS

The elements of the index number problem posed by the seasonality of consumption can now be collected. A point-by-point account reveals an analytical and statistical dilemma on each of the few levels on which some conventional solution to the problem may be sought.

1. In the United States as in other countries (with only very few partial deviations from the common practice), price indexes employ annual rather than seasonal weights. The individual component series of such indexes are price series, each of which would ordinarily reflect the seasonal variation in the given item, except in instances of seasonal discontinuity in pricing when estimates are used instead of reported price quotations. But the weights that serve to combine these series fail to reflect the seasonal variation in quantities consumed or sold. If all intra-annual quantity movements were exclusively seasonal, use of annual weights would be equivalent to the use of seasonally adjusted monthly weights. This, of course, is an extreme and very unrealistic assumption but it nevertheless helps to show that the current price indexes employing annual weights come much closer to being seasonally adjusted than unadjusted so far as quantities are concerned. These indexes, therefore, are of a hybrid

sort in this respect, since the price series of which they are composed are definitely "unadjusted." However, it can also be said that a monthly price index using strictly and exclusively annual weights cannot properly reflect the total seasonal element in the month-to-month change of the average price level precisely because it cannot take account of the existent seasonal fluctuations in quantities.

In the important case of negative price-quantity correlation over the seasons, annual weights may cause some upward bias relative to what would be obtained by application of the proper seasonal weights. For if the price of a commodity is typically lower and its volume traded larger in season than off season, then the average annual quantity weight will understate the importance of price movements during the season (when the price falls to its relatively low levels) and overstate it during the rest of the year (when the price rises to its annual peak levels). This might be regarded as a seasonal variant of the familiar upward bias of Laspeyres indexes (holding true generally in the negative-correlation model). However, such error as may be contributed by the use of annual weights is essentially restricted to monthly within-the-year values and is not expected to distort the series of annual averages; in this, it is different from the year-to-year constant-weight bias which is cumulative.[29]

2. The fixed-base, annual-weight indexes in current use assume a constant "market basket" over a period of time comprehending many seasons—several years. And all conventional price index formulae, including the chain indexes, assume a constant market basket for at least the binary comparison, such as a month-to-month or "inter-seasonal" comparison, since they are designed to measure the change in price of a given household budget supposed to represent a specific level of living. Yet the market basket is not constant from month to month and the existence of commodities that can be priced only in certain parts of the year—the "unique" goods in the seasonal context—makes this fact clear in a particularly forceful way.

3. The Laspeyres and the Paasche chain indexes also fail to satisfy the proportionality criterion which acquires importance in short-term serial comparisons precisely because of the existence of periodic seasonal fluctuations (Section 2 above). Indeed, these basic chain formulae exhibit certain systematic "drifts" over time. By using the Fisher method of crossing the two formulae, it is possible to cope rather effectively with this particular difficulty. But the exceedingly high data requirements posed by all monthly chain indexes with currently changing weights represent a very serious practical handicap.

4. The alternative to the application of varying sets of seasonal weights from year to year (the chain method) is to use a standard

[29] Hence, assuming that the price-quantity relationships are similar in both contexts, there is no doubt that the "type bias" is more serious in its year-to-year than in its temporary, intra-annual form. Since the former error is generally tolerated in the prevailing Laspeyres-oriented practices of index making, it might seem incongruent to make an issue of the latter, noncumulative error. But this is not a very convincing argument. In this writer's view, the size of the error in either form can be only empirically determined. The year-to-year bias, if serious, should not be ignored and neither should the seasonal bias which, even though restricted in time, need not necessarily be negligibly small. Thus it is possible that the negative price-quantity correlation is often more pronounced in the seasonal than in the longer view where cyclical shifts in the demand functions become more important.

Empirical evidence bearing on the issue of the seasonal bias due to the use of annual weights is largely lacking, but some support for the argument in the text above appears in the experiments by Doris P. Rothwell, op. cit., Chart 77A, described on pp. 74–75. This chart shows an index of retail prices for fresh fruits and vegetables showing seasonal movement only and computed by the standard formula with annual weights and by the Rothwell formula with seasonal weights (see Part II, Section 5d). The curve based on the Rothwell formula runs at a significant distance below the annual-weight series during that part of the year when the price is declining and low.

base-year set of seasonal weights over a number of years—as long as the set seems sufficiently realistic. This is a sort of fixed-base approach to the problem of using seasonal weights in price index construction. Such an index presents no insuperable data requirements. Its usefulness, however, depends on how stable the seasonal weight pattern is from year to year. If the intra-annual weight distribution varies considerably over time, the use of a constant set of seasonal weights will result in errors which could possibly offset much of the advantage of having seasonal rather than annual weights.

On the theoretical level, criticism of the present approach centers on the meaning of month-to-month movements in the resulting index series. Since different sets of weights are assigned in this method to each month of the calendar year, comparisons between indexes for different months involve different quantities of the same commodities and even "unique" commodities which are found in one season but not in the other. If a formula that can produce meaningful comparisons of this kind can be devised, it would have an important advantage over the traditional price index measures, including the chain series, which cannot deal with quantitatively and qualitatively different market baskets within a binary comparison. But can such a formula be devised in an operationally as well as conceptually satisfactory way? We shall seek an answer to this question in a comprehensive and systematic survey of methods which constitutes Part II of this study. Meanwhile, let us note that the measurement of the month-to-month change, just like the measurement of the year-to-year change for the chain indexes with seasonal weights, represents the main difficulty for the fixed-base seasonal indexes using a standard set of monthly market baskets. It is already clear that the traditional price index formulae cannot offer a full solution to the seasonal problem, that is, they cannot accomplish simultaneously the following two things: (1) use changing seasonal weights within a year to do justice to the fact that market baskets vary between months (seasons), and (2) provide satisfactory comparisons between the same months of successive years when the market basket (assuming all change to be of the stable-seasonal variety) is constant.

Nevertheless, it would be rash to conclude that the situation is a complete impasse. A critical review of the various possible approaches to the seasonal problem, which is the task we assign ourselves next, should help to identify the possibilities for partial improvements in meeting the problem instead of insisting on a complete solution. It may be anticipated that the familiar types of index formulae will provide some room for such improvements. But in order to make them do so, it will be necessary at certain points to build bridges between the price indexes of the practice and the concepts of the economic theory of cost-of-living indexes. No necessity is seen to accept the contrary view—which seems distinctly unhelpful—that the differences between these two categories are unbridgeable.[30]

[30] There is, of course, nothing new about this position which was often found to be the only logical one to take by students of related index number problems. Thus M. J. Ulmer (op. cit., p. 66) says, in connection with the problem of how to treat new and disappearing goods, that "it is necessary to recall that the fixed budget priced under Laspeyres' formula must be regarded as an approximation to a bundle of goods providing a fixed real income of utility rather than a bill of goods of physically identical commodities. Indeed the very problem of environmental change with which we are dealing testifies that the goal of a physically identical bundle of goods is literally impossible as well as theoretically incorrect."

The full treatment of the problem would require, in addition to these analytical parts, a quantitative estimation of (1) any errors involved in the present procedures; (2) the prospective size of any achievable improvement; and (3) the cost of any improvement. This is too large a program to be carried out with the data and resources available, but Part III of this study will present a considerable amount of materials bearing on some of these issues.

II. Practices and Methods of Dealing With the Seasonal Problem

1. A SURVEY OF THE PRINCIPAL APPROACHES

Perusal of the literature, including descriptions of various price index statistics and suggestions of new procedures, reveals a variety of ways in which problems of seasonality are or can be approached. However, it is possible to bring order into this variety and it will prove helpful to do so. Table IV presents a classification of those practices and methods available for dealing with seasonal variation in consumption that can be represented by, or used in connection with, the conventional price index formulae.[31] The few more drastic departures from the "conventional" are not included in the table, but are treated later in the text. However, the tabulation does include some procedures that are known from other applications but deserve attention for the possibility of being used in the present context, viz., the "substitution" methods (items I, 1 (c), (d), and (e) in Table IV). It also contains two recent proposals designed specifically to cope with the seasonal problem (items II, 1 (b) and (d) in Table IV).

For simplicity, Table IV includes no references to the source of any of the methods or, with respect to the methods used currently or in the past, to the index statistics in which they are or were applied. All this, as well as any other explanation that seemed necessary, is relegated to the text discussion.

The procedures listed in Table IV fall naturally into two groups, those using annual quantities and those using seasonal quantities in weights. Within either group a distinction can be made between the fixed-base and the chain indexes. Fixed-base indexes employing annual quantity figures in their value weights are of course severely restricted in their ability to make any allowances for the seasonality of consumption. Yet if they include any items that are "out-of-season," i.e., that are not traded, during any part of the year, then they must also involve some ways of dealing with such items. These ways may be merely devices to circumvent the seasonality problem but they will not avoid having some implications of their own regarding the behavior or the meaning of the index. It will be shown that these implications vary considerably depending on the nature of the device used. Of course, for a fuller recognition of seasonal variation,

[31] Table IV relies on verbal description rather than on formulae, for two reasons: (1) Formulae would fail to indicate some of the differences between the methods, since the latter may vary only with respect to their treatment of out-of-season items, a matter often considered "outside of the index formulation" (this applies, e.g., to the price indexes of the BLS). (2) Formulae differ according to the precise system of weighting chosen (e.g., Laspeyres, Paasche, etc.) but these are features that need not be specified for the purposes underlying Table IV.

TABLE IV.—*Treatment of the Seasonality Problem in Price Index Numbers: A Conspectus of Methods and Their Implications*

I. ANNUAL QUANTITIES USED IN WEIGHTS

A	B	C
1. Fixed-base indexes: (a), (b): Two methods of estimating price changes for the out-of-season (OS) commodities.	(a) Prices of OS items held constant in off-season period. (b) Price change for OS items assumed equal to that of related year-round items.	Seasonal variation in the market basket (SMB) effectively disregarded. Some SMB implied by treatment of OS items.
(c), (d), (e): Methods of substituting items available in the given season (IS) for and transferring to them the weight of, the OS items.	In these procedures, differences in quality or utility per unit of the substitute items may be: (c) ignored, as when the new priced (IS) item is given the quantity weight of the OS item; (d) assumed to be measured by relative prices of the substitute items, as when the expenditure weight is held constant at the time substitution is made; (e) estimated on the basis of some independent quantitative criteria, e.g., caloric contents of food items.	Attempt to allow explicitly for SMB by item substitution, while holding constant the basic annual weights allotted to certain groups or types of commodities.
2. Chain indexes with annual base period quantities used in weights.	Price change for OS items from the end of one pricing season to the beginning of the next is not reflected in the index.	Attempts to allow explicitly for SMB by linking in and out the seasonal items.

II. SEASONAL QUANTITIES USED IN WEIGHTS

A	B & C	D
1. Fixed-base indexes: (a) Prices for a given month (season) compared with those of same month of the base year. Quantities of the latter period used in both numerator and denominator of the index. (b) Similar to (a) but cumulative: indexes for January, Jan.-Feb., first quarter, etc., are obtained to measure, within any year, the change in the cumulating influence of the seasons. Basic index annual. (c) Prices for a given month and quantities for the same month of the base year used in numerator; expenditures in base year used in denominator of the index. (d) Prices for a given month and quantities for same month of the base year used in numerator; base period annual average prices and same quantities (i.e., those for the proper month of the base year) used in the denominator of the index.	Seasonal quantities are used throughout. Hence, out-of-season items are eliminated from the indexes for the times of the year when they disappear from the market, and the seasonal variation in the market basket is reflected in the indexes.	Month-to-month change in this index is of questionable meaning [it is equal to the ratio of cost of the market baskets in given and previous month of the current year, divided by a corresp. ratio for the base year). No monthly index is provided by (b). Month-to-month change in (c) is simply the current ratio of cost of the market baskets. Month-to-month change in (d) is the current cost ratio divided by a seasonal quantity index.
2. Chain indexes with seasonal base period quantities used in weights.	Seasonal items are linked in and out as they appear and disappear from the market. Index reflects seasonal variation in the market basket.	Year-to-year changes in these indexes give unrealistic results (cf. the proportionality test and drift argument in text).

Key: A—General description of the indexes and procedures.
B—Method of handling the out-of-season (OS) items.
C—The implications or effects of the method with regard to the seasonal variation in the market basket (SMB).
D—The implications of the method with regard to the measurement of month-to-month or year-to-year changes in price levels.

weights appropriate to the compared seasons must be used, and Part II of the table identifies a number of methods employing such weights.

The individual practices and methods summarized in Table IV will now be discussed in a series of critical appraisals. We begin with two procedures used in the principal U.S. price index series and represented by the first two entries in Table IV (items I, 1(a) and (b)).

2. CURRENTLY PREVALENT PRACTICES

a. *Holding Out-of-Season Items Constant.*—Prior to January 1953 the procedure adopted by the Bureau of Labor Statistics with respect to goods which are not sold in certain months of the year (or for which prices are not available at certain seasons even though some trade in them does take place) was to carry such items during these "off-season" periods at the same prices at which they have last been reported. The price change during the off-season months was thus assumed to be zero for any such item; the price was held constant until a new quotation became available in the next season. At this first pricing of a season, the full price change for the given item from the end of the previous season was reflected in the current month's relative and index. The basic nature of the BLS indexes (Laspeyres' fixed base, in our notation $P^L_{0j}=\Sigma p_j q_0/\Sigma p_0 q_0$) was not affected by the above procedure. Since 1953, the BLS partially discontinued this practice in favor of an imputation method which will be described in the next section (b) of this review. But the procedure of holding out-of-season items constant is still followed by the Agricultural Marketing Service (AMS) in all instances of effective seasonal disappearances among the prices received and paid by farmers.

The argument in favor of this procedure is that it does not pretend to do anything that an index using annual quantity weights is not designed to do. Such an index, it may be argued, cannot properly take account of the effects of the seasonal variation in consumption. Efforts to allow for such effects nevertheless, which must resort to some technical devices that would let some of the seasonal elements "slip in through the back door," can at best have only partial success and may be seriously misguided. Hence the best way, if an annual-weight index is used, is to make minimum assumptions in regard to the out-of-season items (which must be dealt with somehow) and to avoid steps which would influence the behavior of the total index in any major or systematic fashion.

Incidentally, the specific difficulty posed by the out-of-season commodities would, of course, be completely avoided if such items were altogether omitted from the market basket; and indeed the practice of excluding them was followed frequently by index makers in various countries, particularly before World War II.[32] However, such omissions obviously reduce the representativeness of an index to an extent which varies for different index measurements but seems in general large enough to be disturbing.

The strongest criticism that has been leveled against the "constant-off-season-price" estimation procedure asserts that the method introduces into the index fictitious prices. The price attributed to an item

[32] Cf. International Labour Office, "Cost-of-Living Statistics, Methods and Techniques for the Post-War Period," report prepared for the Sixth International Conference of Labour Statisticians (Montreal, 4–12 Aug., 1947), ILO Studies and Reports, new series, no. 7, part 4, Geneva, 1947.

included in the index (but not actually traded) in, say, June (e.g., a winter overcoat or leather jacket) may in fact be a February or March quotation.[33]

The charge is harsh but it is only partially true. If the price in June is nonexistent or unknown or entirely unrepresentative and thus unusable, then, given the concept of an index which requires that some price be used for each component of the (unchanged) market basket, there is no escape from estimation of some sort which may be viewed as containing elements of fictitiousness. But keeping its off-season price constant serves to hold down to the minimum the contribution of the given item to the change in the index as a whole. In fact, the method makes this contribution nil in the off-season estimation period and so the question here is really whether this does not overstate temporarily the stability of the index. The answer to this question, however, depends on the prevalent direction of change in prices of possible substitutes for the passive off-season items, which in turn is likely to depend on the general price movement in the given period.

The issue actually gave rise to a complaint about the practical implication of the "constant off-season-price" method. In Congressional hearings conducted under the fresh impact of the strong inflationary movement during the first year of the Korean War, a labor representative pointed out that "in periods of generally rising prices this practice introduces a downward bias into the index. The weight of the seasonal items carried at constant prices exercises a dragging effect on the index so that it does not adequately reflect the rise in prices which is taking place on items being purchased. This downward bias is accentuated by the fact that seasonal merchandise is frequently sold at abnormally low prices at the end of the season. Consequently the constant price at which such goods are carried in the index during the off-season is unrepresentative of the price generally paid by workers during the previous seasons." [34]

Seven years earlier, the Mitchell Committee report of 1944 expressed the belief that "the use of uniform weights for all seasons has tended to cause a downward bias in the index during the past few years." But it continued with the comment that "The introduction of seasonal weights might not be worth the trouble they would involve. . . ." [35]

Again, however, a warning is in order not to overestimate the importance of this point. The just noted "bias" of the index due to the practice of holding the off-season prices constant should be recognized as (a) dependent on the general price movements (if the error is in a downward direction during inflations, it is also in an upward direc-

[33] For a categorical criticism of this type, see Bruce D. Mudgett, "The Measurement of Seasonal Movements in Price and Quantity Indexes," *Journal of the American Statistical Association,* March 1955, p. 93. Mudgett also stresses the inappropriateness of annual weights in the same connection. The argument that weights involving annual quantities do not measure the importance of the index components in the successive seasons is of course right, but we must remember that it applies to annual indexes generally and not specifically to the estimation method now under discussion.

[34] Consumers' Price Index, *Hearings before a Subcommittee of the Committee on Education and Labor,* House of Representatives, 82nd Congress, 1st Session, Statements appended to testimony by Solomon Barkin on behalf of the CIO, U.S. Government Printing Office, Washington 1951, p. 261.

[35] Office of Economic Stabilization, *Report of the President's Committee on the Cost of Living, Appended Report IV* (an appraisal of the BLS index of the cost of living by a committee consisting of W. C. Mitchell, chairman, S. Kuznets, and M. G. Reid, June 15, 1944), U.S. Government Printing Office, 1945, p. 289. The statement quoted at the end of the paragraph in text appears to reflect merely a general impression of the committee; according to our information, no empirical study of the dimensions of the seasonal problem has been undertaken for the 1944 Mitchell report.

tion during deflations); (b) transitory in either case, since the record for any of the affected commodities will always be "rectified" at the first pricing of the given item in its new season. Moreover, this is a procedural effect attributable only to the "seasonal disappearances" which are diffused and not so numerous. Hence any error due to it is likely to be small and overshadowed by other larger and more systematic errors (including the intra-annual upward bias due to the use of annual rather than seasonal quantity weights, which is independent of the direction of the general price-level movement; cf. Part I, Section 4 above).

When the item that has vanished for its off-season period returns to the market, the actual price quotation for it is introduced and the series may undergo a sudden and abrupt shift upward or downward (more likely upward, since in the first month of the new season many items are still scarce and have high prices which then rapidly decline). But this difficulty is shared by the present procedure with its alternative to be discussed next.

b. *Imputing Out-of-Season Items to the Group.*—The current method used in the Consumer Price Index since its revision in 1953 is to assume that the price change for each out-of-season item in a commodity group is equal to the average monthly change in prices of the in-season components of the group (or, in some cases, of its year-round components). For example, the price change for strawberries or peaches from one winter month to another is taken to equal the average price change, in the same monthly interval, of all "fresh fruits" then available. Thus the change of all priced items in the group serves as a basis for estimation of the prices of items out of season. At the first reported pricing of the new season, however, the price change for any item that has just prior to that been so estimated is computed from the end of its previous pricing season. The weight applied to this price change is then also the corresponding end-of-season value. By this means a correction of the previous (off-season) months' estimates is taken and reflected in the current month's index.

An index maker who aims at measuring simply price changes in constant market baskets will describe this procedure as just another "practical solution"—a device to handle the problem of out-of-season commodities within the framework of an index using constant annual quantity weights. This way of viewing it is, of course, perfectly legitimate. However, it is possible to see behind this practical method a principle which can be given an interpretation that is less technical and more economic, less formal and more substantial.

To explain, suppose the rule is adopted that items which fall out of season cease to enter into the calculations but that their (annual quantity) weight is distributed proportionately over the remaining items within the group. It is easy to show that the results thus obtained are the same as those of the imputation procedure described before.[86] These then are two equivalent interpretations of one and the same method. But the second interpretation makes it particularly clear that, while the total weight carried by the group containing the seasonal items remains constant throughout the year, the number of items priced within that group and consequently their effective weights undergo a certain amount of intra-annual variation. When an item such as strawberries becomes unavailable or at least very scarce and

[86] See footnote 86 on p. 261.

expensive, other commodities, say all the fresh fruits not in short supply, are substituted for it. If an intragroup substitution relationship of this kind were empirically established, this would provide a strong rationale for the method in question.[37]

Under the straightforward interpretation of the present procedure as "imputation," there is not much that can be said about it in the way of a general critique. However, if an out-of-season commodity does not disappear completely from the market, yet its price is not available, this should as a rule indicate a pronounced shortage of the given item. The average price change for goods belonging to the same group but available in normal supplies would presumably be a poor indicator of the price change for a good in such short supply. But then to include an item at its very high off season price and its average annual quantity weight (rather than at its much smaller off-season weight) would probably amount to an even worse distortion, which we would certainly wish to avoid. All of which, of course, merely illustrates the basic inadequacy of constant annual weights for combining price changes at different seasons.

This method, too, will often result in large and abrupt price changes at the beginning of a new season when a transition is made from the imputed price to the actual quotation for the "reappearing" item (see Part III, Section 6a below for some numerical illustrations). To be sure, difficulties on this point of transition are unavoidable, since the true situation here involves the presence of "unique" goods which, as was shown before, cannot be handled by any of the customary price index techniques.

If the interpretation of the method as an intragroup "weight-transfer" or substitution is adopted and the assumption of a complete temporary disappearance of the out-of-season items is made, then the

[38] Consider the following illustrative data:

	Annual quantity weight	Prices in period "1"	Prices in period "2"
Year round items:			
A	45	5	7
B	30	3	4
Seasonal item: S	25	6	0

The price index for A and B is $P^L_{A,B}=\frac{7(45)+4(30)}{5(45)+3(30)}\times100=138.1$

The price index for A, B, and S, according to the "first interpretation", is

$$P^L_{A,B,S}=\frac{7(45)+4(30)+6(25)P^L_{A,B}}{5(45)+3(30)+6(25)}\times100=P^L_{A,B}=138.1$$

$$\left(\text{In general: If } P^L_{A,B}=\frac{a}{b},\text{ then } P^L_{A,B,S}=\frac{a+\frac{a}{b}c}{b+c}=\frac{a}{b}\right)$$

According to the "second interpretation," S disappears from the index for period "2" since its quantity and price are then both zero, and the weights of A and B become, respectively, $45+\left(\frac{45}{45+30}\right)25=60$ and $30+\left(\frac{30}{45+30}\right)25=40$

The index calculation is now:

$$\frac{7(60)+4(40)}{5(60)+3(40)}=P^L_{A,B}=P^L_{A,B,S}=138.1$$

[37] To be sure, the method has not in fact been given any such basis. The index maker did not seek to do this; moreover, if he did, he would have come up with some other combination of substitutes closer to reality, since the simple intragroup relation implied is not really plausible.

analytical situation is in principle clearer. Ideally, one would want to find one or more (i.e., a combination of) perfect substitutes for any item that drops out and transfer to them the weight of the item, taking proper account of the relative amounts of "utility" or service provided per unit of the substitute commodities. It is obvious that this is merely a conceptual standard of perfection which cannot even be closely approximated in practice, let alone attained. Substitutes are virtually always more or less imperfect, and the degree of substitutability eludes measurement. Even so, a careful, explicit attempt to use selected plausible combinations of substitutes would not be unlikely to lead to an appreciable improvement over an implicit substitution procedure via weight transfer within a single narrowly defined group. Thus, the current BLS practice of imputation uses only fresh fruits as, in effect, a group of substitutes for each of the seasonal fruits such as grapes or watermelons (there are five such items in the CPI). But there is no reason why canned and frozen fruits should be excluded from the role of substitutes for seasonal fresh fruits, where one would expect them to be, on the contrary, quite important.[38]

3. METHODS OF SEASONAL SUBSTITUTION

Seasonal changes in the composition by commodities of the market basket can to some extent be handled explicitly even within the rigid structure of a fixed-base index using annual quantity weights. Common to the methods that fall under this heading is the central idea of seasonal substitution, which has already been introduced in the preceding section's discussion of the weight-transfer procedure.[39]

a. *Direct Replacement of One Price Series by Another.*—In this approach, which in Table IV is listed under the category I, 1 as item (c), the current month's price of a new article B is directly compared with the previous month's price of the article A that is being replaced. The quantity allotted to B is the same as that previously allotted to A.

[38] For evidence supporting this expectation, see Part II, Section 4 below. The above suggestion concerning canned fruits has a counterpart in the practice currently adopted in Britain for the new Index of Retail Prices of the Ministry of Labour and National Service, which replaced a postwar "interim" index as from January 1956 (except that there the substitutes are fresh and canned vegetables; the same role was proposed for fruits, but presently the British index includes no fruits that are not priced throughout the year). Cf. Great Britain, Ministry of Labour and National Service: (1) Cost of Living Advisory Committee, "Report on the Working of the Interim Index of Retail Prices," Cmd. 8481 (March 1952), p. 31, § 78; (2) "Method of Construction and Calculation of the Index of Retail Prices," Studies in Official Statistics: No. 6, London, H. M. Stationery Office, 1956, pp. 13–14.

It may be noted that otherwise the U.K. practice with respect to the out-of-season commodities is in effect the same as that currently used in the U.S. Consumer Price Index. While the U.S. method is interpreted as an "imputation," the U.K. method is interpreted as a "weight transfer," but as we have seen these are two readings of the same procedure. The official description of the British procedure (see ref. 2 above) is cast in technical terms and does not refer to any theoretical considerations regarding substitution between the seasonal and the year-round commodities. It does point out that some intra-annual variation in "effective weights" results from the application of the method, but stresses that the latter is designed as a practical device for "adhering as closely as possible" to the conception of pricing a fixed market basket during periods of temporary unavailability of some of the items included in the index. This position is analogous to that taken by the U.S. index makers.

[39] Substitutions are used in price index measurements primarily in connection with changes in the quality of goods produced and consumed at different times. The methods discussed in this section have all been so employed, in either some regularly published or some proposed index series. Although the seasonal problem differs from the quality problem in several respects, the two do have certain features in common and it will be instructive to review these methods with reference to their applicability to the issue of seasonal variation in the market basket.

The two are simply treated as if they were one and the same item; any differences between them in quality or "utility" are disregarded. The ratio p_2^B/p_1^A measures the price development at the time of the substitution assumed to occur between the periods 1 and 2.

The method is used by the BLS only for substitutions within narrowly defined specification ranges. A less restricted form of employing it, with rather different implications, would consist in direct comparisons of the *average prices* of a number of varieties of a product; this is a procedure designed to handle changes in the available assortment of such varieties, which often reflect changes in the product's quality.[40] But it is clear that, even in its most relaxed form, this method is largely inadequate in the seasonal context, since the goods substituted for each other because of seasonal changes in the demand and/or supply conditions differ in various ways, and the differences between them are as a rule too large to be neglected. We know of only one instance in which a variant of this method is applied to a seasonality problem, namely, the case of tobacco in the Index of Prices Received by Farmers. There an average price is computed each month for the types of tobacco that are actively being sold and the most recent season average for types not currently sold.

b. *Price Ratios as Measures of Relative Utilities.*—In this approach, price relatives, p_2^A/p_1^A and p_3^B/p_2^B, are used as measures of price change from period 1 to 2 and from 2 to 3, respectively, and their product is the measure of the price development between the periods 1 and 3. This method presupposes that A and B are available for sale and priced simultaneously at least during one unit period. The technical consideration behind it is that the movement of the index in either the 1–2 or the 2–3 interval will not reflect any differential in price that is due to differences in quality between the two articles, only the price change proper in either A or B. But this, while pertinent, is clearly not a sufficient argument in favor of the method. The shift from A to B is not a mere technicality and it will hardly fail to influence the behavior of the index.[41] The really important consideration, then, is what is used to replace what and when.

The method implies that at the time the substitution is made the consumer spends the same sum of money on B that he or she used to spend on A. The consumer may be presumed to derive equal utility per dollar from either purchase. This means that the relation between the qualities of the substitute goods is taken to be measured

[40] The price development would here be measured by a ratio of the form $\bar{p}_2/\bar{p}_1$, where the bars refer to the averaging operations and the superscripts to the combinations of the varieties included. (For example, $\bar{p}''_2$ may be the average in period 2 of prices of A, B, C, and E; $\bar{p}'_1$ the average in period 1 of prices of A, B, C, and D.) The procedure may work well with regard to certain product categories for which optimum specification ranges can be found. These should be broad enough to include all varieties of the product that are close substitutes in terms of the pertinent quality standards. If there are enough such varieties, the number and quality of price quotations for them may be adequate to yield stable and representative averages to be used in the calculation of the price relatives. But such favorable situations are probably not very frequent even in the content of quality changes for which the method has been considered. The specification ranges may be too narrow to provide useful averages; or it may prove impossible to restrict them properly without application of more refined methods because of their multidimensional nature (cf. Andrew T. Court, "Hedonic Price Indexes With Automotive Examples," *The Dynamics of Automobile Demand*, General Motors Corporation, New York, 1939, pp. 103–105).

[41] Suppose that A is still available for pricing in periods 3, 4, etc., then the index would presumably move differently if it continued to carry the old article instead of the new item B; and its behavior would be different again if the substitute were C rather than B.

by the ratio of their prices. If B has y times the quality of A, then implicitly the quality ratio y is assumed to equal p_2^B/p_2^A.

It is of course an elementary proposition of the demand theory that the equality of the corresponding price ratios and marginal utility ratios, or marginal rates of substitution, defines the maximum (equilibrium) position of the consumer. This makes the present approach, which implies that the price ratios tend to reflect the y-values (taken to express the "relative qualities" of the substitute goods as seen by the consumer), highly appealing on *a priori* grounds in all those situations in which consumer choice can be viewed as generally free and informed and in which the market mechanism can be assumed to enforce the necessary adjustments of relative prices at some time during the transition from one variety or set of items to another. By the same token, where the above conditions are *not* fulfilled, the method lacks its theoretical basis. This is clearly the case when the transition from one product to another is sudden rather than successive, decreed by authorities or imposed on households by some external factors rather than due to voluntary actions of consumers under market influences.

A few factors can be listed in favor of applying this method (listed as item I, 1(d) in Table IV) to the seasonal problem. Transition to other products from items that have become seasonally scarce is as a rule gradual; seasonal goods do not disappear from and reappear on the markets overnight but rather their availability decreases or increases successively over a number of weeks or a few months at certain times of the year. Substitutions due to seasonal change are expected and essentially "voluntary." The element of the new and as yet unfamiliar, which is often a complicating factor in the quality context, is here negligible.

On the negative side, it is not so much any inherent unfavorable features of the problem that stand out as a likely source of difficulties but rather certain technical limitations of the method in its practical use. It is natural to think of the present method as applicable primarily to *paired* substitutes. But in the approach to the seasonal problem, as in many other applications, restricting the substitution to two commodities at a time strikes one as artificial and scarcely satisfactory. Relatively few goods have single specific substitutes; most by far have several or many partial substitutes which, moreover, need not be restricted to any easily defined commodity group. Thus decreased consumption of strawberries in the winter may be replaced by increased consumption of, say, pudding or jellies as well as of apples or bananas.

In principle, the situation can be dealt with by forming groups of related goods and making the substitutions within such groups; in practice, to be sure, this task would be far from easy. However, if we can judge it from the present contents of the principal U.S. indexes, the items that effectively disappear from the market at certain seasons

As applied to quality changes, this procedure will usually not directly affect the level of the index. In practice, prices of strictly specified articles are not frequently changed; rather such price variation as occurs here typically accompanies the introduction of a new brand, make, or model of the product. The above method will then amount in effect to a multiplication of two price relatives which, however different their components, are each equal to one (i.e., since $p_1^A=p_2^A$ and $p_2^B=p_3^B$, $p_2^A/p_1^A \times p_3^B/p_2^B=1$). Thus Hofsten (op. cit., pp. 49 ff.) treats this method as if it always gave an index equal to unity. In connection with seasonality problems, however, it is well to remember that either or both price relatives involved may differ from one and so may, consequently, the result of this substitution procedure.

are not very numerous or complex, so that specific substitution schemes to deal with the special problem of these disappearances might prove practical. In fact, the imputation procedure currently used by the BLS (item I, 1(b) in Table IV discussed in Section 2b above) has all the formal characteristics of the group substitution method just referred to. It does, however, differ from that method in one material respect: as was argued before, the BLS device is "substitution" more by implication than by design. A full-scale attempt to apply the present method explicitly would have to involve careful studies designed to determine (1) in what areas of the index groups of related items should be distinguished for purposes of seasonal substitution; (2) what items should be included in any such group, so as to minimize the intergroup dependence and substitution; and (3) what is the optimal timing for any substitution to be made.[43]

c. *Independent Estimation of Relative Utilities.*—It has been suggested that criteria other than relative prices be applied to comparisons of the substitute goods. In this view, the previously introduced "quality ratio" q should be measured "objectively" on the basis of the serviceability of the goods in question, leaving out of consideration "factors which are not of real utility." [44] But it is clear that for many products "quality" as a sum of objective properties of a given article, as distinct from its subjective valuation, is not a very pertinent concept so far as our problem is concerned. For example, caloric content measures are available for foods, but any food item has properties other than caloric content that are important to consumers and hence cannot be disregarded. Quality itself is usually a composite, and so it might be argued that the solution lies in taking proper account of all of its essential elements rather than selecting arbitrarily one, such as, for a food product, its caloric value. But the basic difficulty here is not so much with the *number* of the relevant characteristics of a given good as with their *nature*. It is likely that closer relationships will be found between the prices and the objective quality characteristics for certain complex items of machinery than for many simple food or apparel products, simply because, in the evaluation of the former, objective measurable properties play a decisive role while, in the evaluation of the latter, individual subjective taste factors are particularly important. Now for those products with close and stable price-quality relations the problem of dealing "objectively" with quality differences is, of course, in principle manageable, even though it may be in practice highly complex because of its multidimensionality.[45] But as it happens, few of the seasonal goods seem to belong to this category and many to the other one that does not favor the "objective" approach. The usefulness of the latter in regard to seasonal substitutions is therefore believed to be very limited.

[43] Certain simple rules could be tentatively adopted in these empirical inquiries. For example, dovetailing consumption seasonals might be taken as *prima facie* evidence of a significant seasonal substitution. Months of extremely low consumption may be excluded or at least avoided in the selection of the proper timing for the linking-in operations.

[44] E. v. Hofsten, op. cit., p. 120. It may be noted that some of these arguments (Hofsten's approach is a good example) seem to be based on questionable generalizations which lack firm analytical support: views that the commodity world has grown so complex that consumers can have no adequate knowledge of "articles for sale," that advertising often persuades the public to buy goods that will fail to give them the satisfaction which they expected, etc. If the consumer is not irrational or ignorant, then he is the best arbiter of the values to himself of the commodities or services acquired; if he is, then how is anyone else to tell what the "true" values for him are?

[45] An ingenious approach to this problem was suggested in 1939 by Andrew T. Court, op. cit.

An interesting example of a "quality" adjustment that is being made in the seasonal field is offered by the new British Index of Retail Prices. In this index, the difference in price between new potatoes and old potatoes (from the previous year's crop) is considered to reflect in a certain degree a change to a higher-quality article. To allow for this change, 5½ lb. of new potatoes are taken as equivalent to 7 lb. of old potatoes in July. In mid-August the ratio used is 6 : 7, and in mid-September 6½ : 7; thereafter no further adjustment is made.[46] The decline in the ratio apparently reflects the fact that as the season progresses the "newness" of the current year's potatoes wears off: they become more plentiful and less expensive relative to the old crop which they soon replace. Thus the adjustment is not based merely on objective quality differentials, which remain constant during the transition period, although the official description of the index stresses the fact that new potatoes generally involve less wastage and possess a higher nutritive value than old ones. Rather, the procedure has the merit of taking into account, to some extent, the changing market positions of the two items.

4. CHAIN INDEXES WITH ANNUAL BASE PERIOD QUANTITIES IN WEIGHTS

No conventional index formula employing basically *annual* weights can be really satisfactory in dealing with the seasonal problem. This applies also to the "chain method" insofar as the latter retains base period weights (item I, 2 in Table IV). This is not a "true" chain because it is based on fixed period weights rather than on changing weights for the months being compared. It is, however, not identical with the corresponding index series calculated by the fixed-base method because we conceive this chain as being confined to items common to the two successive months being compared, so that the price change for out-of-season items from the end of one pricing season to the beginning of the next one would not be reflected in the index. But if seasonal disappearances occur, then this principle is sure to introduce such differences between the successive links of the chain as to make the chain series diverge from the corresponding fixed-base series. In this case, our chain index will fail the proportionality test just as any true chain index would, and, as shown in Part I, this is a serious weakness as far as the seasonal problem is concerned; and at the same time we will not even have the advantage that a full-fledged chain index would give us, for we will not have utilized all the information on the weights for the successive months or seasons. Hence no gain is seen in this rather halfhearted use of the chain method.

5. FIXED-BASE INDEXES WITH SEASONAL QUANTITIES USED IN WEIGHTS

It is natural to seek a more complete and satisfactory solution to the seasonal problem by devising methods involving seasonal weights and working out their implications. There is of course no difficulty in measuring the average price change between the same months of successive years, if a month is our unit "season," and if a constant seasonal market basket can be used, for traditional methods of price index construction can be applied in such comparisons. For each month of the year, a list of commodities representative of consumption in the given month would have to be made up, specifying the quantities

[46] For more detail, see pp. 12–13 in the Ministry of Labour 1956 pamphlet cited in footnote 38.

purchased of each item. The resulting 12 seasonal market baskets for the Januaries (J), Februaries (F), etc., may be represented by as many column vectors of quantities $\{Q\}_J$, $\{Q\}_F$, . . . Q_D. The current dollar value of, say, $\{Q\}_J$ in the "first" year (to be denoted by the subscript 1) would be $[P]_{J1}\{Q\}_J$, where $[P]_{J1}$ is a row vector of appropriately dated prices of items included in the January market basket. The expression for the year-to-year change (say, between the Januaries of years "1" and "2") can now be written in two equivalent forms, first using the simpler vector notation and then using the traditional notation, to read

$$\frac{[P]_{J2}\{Q\}_J}{[P]_{J1}\{Q\}_J} \text{ and } \frac{\Sigma p_{J2}q_J}{\Sigma p_{J1}q_J} = \frac{\Sigma \frac{p_{J2}}{p_{J1}} p_{J1}q_J}{\Sigma p_{J1}q_J}.$$

This is in itself a satisfactory formula for a "binary comparison" on a seasonal basis, judging by standards of the classical or orthodox price index theory, which are widely accepted in index making. The basis for the seasonal quantities could be changed, if it were so desired, to satisfy a Paasche-type or some other formula. While all this is simple enough, the real difficulty that must now be faced is how to construct an index on the base period which would (a) be consistent with the above form for year-to-year comparisons and (b) imply also an acceptable measure of the average price change from month to month. It will be shown that these requirements cannot be easily or completely satisfied.

a. *Comparisons with Same Month of the Base Year.*—The procedure that suggests itself most readily is to compare the prices for a given month with those for the same month of the base year, using quantities for the latter period in weights (see item II, 1(a) in Table IV). Let us use the subscript j to denote a given month of the year and the subscripts 1,2, . . . to denote the years 1,2. . . . Using 0 to identify the base year, the index on the base period for the month j of, say, year 2 is then

$$\frac{[P]_{j2}\{Q\}_{j0}}{[P]_{j0}\{Q\}_{j0}} \text{ or } \frac{\Sigma p_{j2}q_{j0}}{\Sigma p_{j0}q_{j0}} = \frac{\Sigma \frac{p_{j2}}{p_{j0}} p_{j0}q_{j0}}{\Sigma p_{j0}q_{j0}}.$$

This formula differs from that given previously for the year-to-year comparison only in that the price vector in the denominator refers now, not to the same-month year-ago period, but to the same-month-of-the-base-year period. (The subscripts of the quantity terms are here $j0$ because the base period and the weight period are taken to coincide; they would also apply to the previous formula under the same construction.)

By dividing the above index number into that for the next month, the measure of the month-to-month change implicit in the present formula is shown to be

$$\frac{[P]_{j+1.2}\{Q\}_{j+1.0}}{[P]_{j+1.0}\{Q\}_{j+1.0}} \div \frac{[P]_{j2}\{Q\}_{j0}}{[P]_{j0}\{Q\}_{j0}} = \frac{[P]_{j+1.2}\{Q\}_{j+1.0}}{[P]_{j2}\{Q\}_{j0}} \div \frac{[P]_{j+1.0}\{Q\}_{j+1.0}}{[P]_{j0}\{Q\}_{j0}}.$$

This result, especially in its second form, has a meaning that can be verbalized. But this meaning is not simple; the formula does not represent a direct measure of the average price change between the two months, and translating it into words cannot, of course, change this fact. What the formula offers is really a comparison of two cost ratios: (1) the ratio of the cost of the market basket assigned to month $(j+1)$ at current prices to the cost of the market basket assigned to month j at last month's prices, and (2) the ratio of the expenditures in month $(j+1)$ of the base year to expenditures in month j of that year (i.e., the original cost ratio for the two baskets).

b. *A Cumulative Within-the-Year Index.*[47] In his 1955 article in the J.A.S.A. (see reference in footnote 33) B. D. Mudgett proposed an index using seasonal weights which would differ from the formula just discussed essentially in that it would employ a process of intra-annual cumulation of the monthly value aggregates. In Mudgett's notation, the index on the base period (year 0) for, say, February of some given year i reads

$$P_{0\cdot i2}=\frac{\sum_{j=1}^{2}\sum_{t=1}^{N_{aj}} p_{ij}q_j}{\sum_{j=1}^{2}\sum_{t=1}^{N_{aj}} p_0 q_j}$$

where j (months)$=1,2, \ldots 12$; t (commodities)$=1,2, \ldots N$; N_{aj}=number of commodities in list for month j of the weight year a; P_0=base year average price of commodity t.

This index, unlike the previous one, uses in its denominator the average annual prices, rather than the seasonal prices, of the base year. Assuming again, for simplicity, that the weights of the base year are used $(a=0)$, the only other difference between the two indexes results from Mudgett's use of the cumulation device, which is a specific feature of his formula. Thus, from February of any year on, the formula refers, not to a single month, but to periods of 2, 3, . . . 12 months. The cumulated values are the products of our P and Q vectors for the successive months within each year.

It is easily seen that in Mudgett's formula the December index for any year, as an end result of the intra-annual cumulation process, is identical with the given year's index. The yearly index is the basic index and is of the conventional sort; in the expository formulae of Mudgett's paper it is presented as a fixed-weight aggregative but, as noted by the author, it could be easily rewritten to give the formula of Laspeyres or Paasche, etc. Neither would the question of whether to use a fixed-base index or a chain of annual indexes be prejudged by Mudgett's method of dealing with the monthly changes.

Mudgett claims that his monthly within-the-year indexes "can give an accurate measure of the cumulative influences of price change . . . throughout the months of the year, compared to the corresponding months of the year chosen as base; and this is done with the complete realism that is associated with the disappearance of some commodities

[47] Listed as item II, 1(b) in Table IV.

at some seasons and their reappearance at others."[48] But his method has neither a better nor a worse claim of this sort than has the simpler method described in the preceding subsection (a), except that here the word "cumulative" is in order and there it is not. Actually, neither method provides us with a monthly price index proper. If anything, the meaning of the monthly change in Mudgett's index (evaluated as usual in terms of the ratio of two adjoining index numbers, $P_{0.i,j} \div P_{0.i,j-1}$) seems to be less clear than the meaning of the corresponding measure for the other index. This is due to the cumulation procedure adopted by Mudgett, which does anything but help in the already difficult task of interpreting monthly changes in an index with monthly varying market baskets and weights. It is true that this procedure has its own rationale in that it establishes a link between the monthly index numbers, which are treated as subsidiary, and the annual index, which is regarded as being of central importance. But while some link between the monthly and the annual indexes is certainly necessary, it is not at all obvious that it must have this particular form, i.e., that cumulation cannot be avoided and a more regular time series of monthly price indexes with seasonal weights cannot be constructed. And since monthly measures of average price change, if reasonably satisfactory, can be really useful and are undoubtedly an object of public demand, a proposal that does not provide for such measures is definitely at a disadvantage vis-à-vis others that would improve them.

c. *An Index of Seasonal Variation in Expenditure.* The Canadian Consumer Price Index, 1949 to date, which was introduced in 1952 to replace the old Cost-of-Living Index, uses a particular formula with seasonal weights for a subgroup of food items (item II,1(c) in Table IV).[49] Let P_{ji} be the seasonal food index for month j of year i; year 0 be the base and weight period, and N be the number of commodities (t) on the list for the period indicated by a subscript. Then the formula is

$$P_{ji} = \frac{\sum_{t=1}^{N_{ji}} p_{ji} q_{j0}}{\frac{1}{12} \sum_{j=1}^{12} \sum_{t=1}^{N_{j0}} p_{j0} q_{j0}}$$

The numerator of this expression is equivalent to that of the first index reviewed in the present section, which can be seen directly by rewriting it as $[P]_{ji}\{Q\}_{j0}$. The denominator is the sum of the value aggregates $[P]_{j0}\{Q\}_{j0}$ over the twelve months of the base year, divided by 12. This summation and averaging account for the entire difference between the two indexes. (It will be recalled that in our first seasonal index the denominator was the aggregate $[P]_{j0}\{Q\}_{j0}$ for the appropriate month.)

If the index P_{ji} is divided into the next month's index $P_{j+1,i}$, the denominators of the two, which are equal, cancel each other and the

[48] Bruce D. Mudgett, op. cit., p. 98.

[49] Government of Canada, Dominion Bureau of Statistics, Department of Trade and Commerce, *The Consumer Price Index, January 1949–August 1952* (including an explanatory statement), Ottawa, 1952, pp. 14–15. The group of seasonal foods is composed of fresh and canned fruits and vegetables, fats, eggs, and meats, and poultry. It accounts for 61 percent of all foods.

resulting ratio, a measure of the monthly change implicit in the present formula, may be written as

$$\frac{\overset{N_{j+1,i}}{\Sigma} p_{j+1,i} q_{j+1,0}}{\overset{N_{ji}}{\Sigma} p_{ji} q_{j0}} \text{ or } \frac{[P]_{j+1,i}\{Q\}_{j+1,0}}{[P]_{ji}\{Q\}_{j0}}$$

This is again the ratio of the cost of the market basket appropriate for month $(j+1)$ at current prices to the cost of the market basket appropriate for month j at that month's prices. As such it is identical with the first half of the corresponding measure for our first seasonal index (see text and formula in Section 5a above). In that measure the current cost ratio was taken in relation to the ratio of market basket expenditures for the corresponding months of the base year; here it stands by itself. These two formulae, then, are rather similar, but the explicit reference to the "base season" in the first of our seasonal indexes can be regarded as a point in its favor.[50]

It may be added that seasonal weights are applied only within a single group of foods in the Canadian index, and that this group as a whole, like all the other groups in the index, is assigned a constant annual weight. In this case, then, an internal distribution of weights is being varied from month to month during the year in such a way that seasonal declines in the importance of some items are always exactly balanced off by seasonal increases in the importance of other items, with the combined weight of both categories remaining constant. This can be regarded as a group substitution similar in principle but, to be sure, much more complex in practice than the substitution with proportional weight redistributions discussed earlier in Section 3 of this survey.[51]

d. *Value Ratio Deflated by a Seasonal Quantity Index.*—Recently a new seasonal index method (listed as item II, 1(d) in Table IV) was developed by Doris P. Rothwell in her article in the March 1958 issue of the *J.A.S.A.* to which we have already referred. Rothwell's index on the base period, in the conventional and vector notation, respectively, is appealingly simple:

$$\frac{\Sigma p_{ji} q_j}{\Sigma \bar{p}_0 q_j} \text{ and } \frac{[P]_{ji}\{Q\}_j}{[\bar{P}]_0\{Q\}_j}$$

Here again j is a given month of the year, i is a given year, and 0 is the base year (if base and weight periods coincide, the subscripts of the quantity terms are $j0$).

[50] D. P. Rothwell, op. cit., p. 69, describes the Canadian index as "actually a ratio of expenditures, in which the numerator is the product of monthly quantities and monthly prices and the denominator is 1/12 the annual value weight (or 1/12 the sum of monthly expenditures) in the base year." This is partly incorrect as the quantities in the numerator of the Canadian index refer to the j-th month of the base, not of the current year (cf. the formulae given in the Canadian source identified in footnote 49 above and in the Rothwell article). Nevertheless, Rothwell is right in saying that "some of the month-to-month fluctuation is due to differences in physical quantities" but only provided that the differences referred to are those between the monthly market baskets on which the index is based.

[51] The Technical Committee appointed in 1956 to make recommendations for the new British Index of Retail Prices considered the desirability of internal weight distributions for the fruit and vegetables sections of that index but concluded that these variations in weighting would have little effect and did not advise the use of such a method. Instead they did advise the simpler substitution procedure to which reference was made in Part II, Section 2b (for source see footnote 38).

The value $\bar{p}_0$ is the annual average price in the base year, obtained by weighting the monthly prices by seasonal quantity weights. $[\bar{P}]_0$ is the vector of these p_0 values.

The formula has the merit of yielding the logical measures of price change between the same months of successive years (the measure presented early in this section). But all of our seasonal-weight indexes have this advantage; any index of this type will have it, provided that for a given month its base period calculated denominator is the same in any year.

The Rothwell index also shares with the other indexes some other points that have been advanced in its favor, such as the ability to use the proper seasonal weights and changing commodity lists. Its further advantage is that a weighted average of its monthly values for any given year yields a proper annual index for that year, but again this is not a unique feature of this index.[52]

Decisive for the evaluation of this as well as other seasonal-weight, fixed-base indexes is how they measure the month-to-month change in prices, for this is where the main difficulty for these indexes lies. The ratio of base period indexes for two consecutive months in terms of the Rothwell formula provides an expression for the monthly change that has a particular meaning. We can write it in our two notations as

$$\frac{\Sigma p_{j+1,i}q_{j+1}}{\Sigma p_{ji}q_j} \div \frac{\Sigma \bar{p}_0 q_{j+1}}{\Sigma \bar{p}_0 q_j} \text{ or } \frac{[P]_{j+1,i}\{Q\}_{j+1}}{[P]_{ji}\{Q\}_j} \div \frac{[\bar{P}]_0\{Q\}_{j+1}}{[\bar{P}]_0\{Q\}_j},$$

Rothwell says that "In this form, the price index is an expenditure ratio divided by the quantity index calculated for the base year, or adjusted for the difference in quantities in the two periods."[53]

Since the q-terms refer, not to the actual quantities marketed in the given months of the given year, but to fixed quantities used as weights, the first of the two expressions used in the division is not really an observable "expenditure ratio" but rather a ratio of costs, in the given and the previous month, of certain predetermined baskets of goods. The second expression is a true seasonal index of quantities with average base year prices used as fixed weights. There will be as many such "seasonal adjustment factors" as there are "seasons" distinguished, e.g., 12 in the case of an index with monthly seasonal weights. Thus it is believed that the notion of an adjustment for the seasonal change in quantities fits Rothwell's measure better than her other notion, that of a ratio of an expenditure index to a quantity index. This is better than if the reverse were true, because a division of a value ratio proper by a quantity index need not in all cases yield an acceptable price

[52] The weights that will give the desired result for the Rothwell index are quantity indexes of seasonal consumption $\Sigma q_j \bar{p}_0 / \Sigma q_0 \bar{p}_0$. The annual index obtained can be written as

$$\sum_{j=1}^{12} \sum_{t=1}^{N_{jt}} p_{jt}q_{j} / \sum_{t=1}^{N_{t0}} \bar{p}_0 q_0.$$

(The value q_0 in these expressions is the annual base quantity weight or the sum of the monthly seasonal weights.) By applying to the monthly values of our first seasonal index the ratios between the base period calculated seasonal and annual values, $\Sigma q_j p_{j0} / \Sigma q_0 \bar{p}_0$, the same annual index can be computed.

[53] Rothwell, op. cit., p. 72.

index.[54] But then what is being accomplished by the Rothwell adjustment method is itself open to question, too. It may be argued that, while seasonal weighting is used in the formula, the effect of it is largely canceled again by the adjustment, so that the measure we get does not really reflect the influence of the consumption seasonals or does so only to some unknown but presumably small extent. This indeed may explain why the results of an experimental application of the Rothwell formula differed but relatively little from the results obtained by applying, to the same body of test data, conventional annual-weight methods such as those now and previously used to deal with the seasonal problem at the Bureau of Labor Statistics.[55]

6. CHAIN INDEXES WITH SEASONAL BASE PERIOD QUANTITIES USED IN WEIGHTS

These indexes (item II, 2 in Table IV) have been given sufficient attention in Part I of our study. It will be recalled that the major objection to these formulae is that they tend to produce errors in the range of the year-to-year comparisons.

7. INDEX NUMBERS BUILT FROM SEASONALLY ADJUSTED PRICES AND QUANTITIES

The methods discussed so far did not use any explicit adjustments for the seasonal variation in prices or quantities but aimed at the construction of improved *unadjusted* index numbers. (The resulting series could, of course, be subjected to some seasonal correction procedure.) A treatment of seasonal commodities which would require estimation of the seasonal variation in prices has recently been suggested by Richard Stone.[56]

The method involves the assumption that normal seasonal patterns in prices appropriate to the base year exist and can be expressed by sets of seasonal multipliers, one set for each commodity. A multiplier for the j-th season, say s_j, would thus satisfy the relation $p_j = s_j p^*_j$, where p_j is the actual price of the given item in the j-th season and p^*_j is its corresponding adjusted price. The adjustments must, of course, cancel out over the entire seasonal cycle, i.e., normally over a year; if there are m seasons, then

$$\sum_j^m s_j/m = 1 \text{ and } \sum_j^m p_j = \sum_j^m s_j p^*_j = \sum_j^m p^*_j.$$

[54] Rothwell (ibid., pp. 71–72) states that the "basic idea [of deriving a price index by dividing a value index by a quantity index] is inherent in the formula proposed by the German mathematician, M. W. Drobisch, in 1871 for measurement of changes in exchange values:

$$\frac{I_2}{I_1} = \frac{\Sigma q_2 p_2}{\Sigma q_1 p_1} \cdot \frac{\Sigma q_1}{\Sigma q_2}.$$

This, however, contains the unrealistic condition that the quantities must be expressed in the same units so as to be additive."

The Drobisch formula is indeed a poor precedent, especially as far as the requirements of the seasonality problem are concerned, and not just because of the additivity issue. As noted in Bortkiewicz, *Nordisk Statistick Tidskrift,* III, 1924, pp. 510–512, the Drobisch formula does not satisfy the proportionality and identity tests. It is at least questionable whether it should be regarded as a price index proper. Let us add that the unweighted quantity index, $\Sigma q_1 \Sigma / q_2$, can be replaced as the divisor in the above formula by a weighted quantity index, e.g., $\Sigma q_1 p_0 / \Sigma q_0 p_0$ or $\Sigma q_1 p_1 / \Sigma q_0 p_1$. In these cases, the results are simply a Paasche or a Laspeyres index, respectively.

[55] See D. P. Rothwell, op. cit., pp. 74–75, and Chart 77B on p. 77 (also the statistical tables in the appendix available upon request to the author).

[56] R. Stone, *Quantity and Price Indexes in National Accounts,* Organization for European Economic Cooperation, Paris, 1956, Chapter VI, particularly pp. 74–77.

Stone then suggests that the adjusted quantity measure in terms of which different seasons can be compared is $q^*_j = s_j q_j$. For any given good, then, the seasonal multiplier for quantity is the reciprocal of the corresponding seasonal multiplier for price. This can be viewed as an implicit assumption of unitary elasticity of demand in the seasonal context: if in an off-season month the price is, say, 10 percent higher than it is on the average during the year, the quantity consumed is presumed to be 10 percent less than its mean annual per-month rate. Alternatively, one may regard this treatment as a substitution for the physical quantity units of a system of measurement in what may be termed the "equivalent-seasonal-value" units. For example, one may decide that "a product quantity of the December variety should be reckoned as equivalent to twice as much of the June variety."[57] In contrast, physical units such as pounds or barrels are said to be not comparable between the seasons because they do not take into account the "seasonal quality differences."[58]

In these terms, Stone's formulation of an annual Laspeyres quantity index for a single commodity with several "seasonal varieties" is simply

$$\sum_j^m q^*_{j1} p^*_{j0} / \sum_j^m q^*_{j0} p^*_{j0} = \sum_j^m q_{j1} p_{j0} / \sum_j^m q_{j0} p_{j0},$$

where the subscripts 0 and 1 denote, respectively, the base and the current year. The equivalence of the two expressions reflects the fact that price and quantity adjustments are in this approach designed to cancel out for each season, leaving the values unchanged $({}_jpq_j = p^*_j q^*_j)$. The formula

$$\sum^m q_{j1} p_{j0} / \sum^m q_{j0} p_{j0}$$

could also be obtained by treating the supplies of a product that are available in different seasons as different commodities and averaging the quantity changes between the base and the current year for each season separately, using as weights the proper seasonal expenditures.[58]

By defining the mean adjusted price and the mean adjusted quantity in the base year, respectively, as

$$\bar{p}^*_0 = \sum_j^m p^*_{j0} \overset{*}{q}_{j0} / \sum_j^m \overset{*}{q}_{j0} \text{ and } \bar{q}^*_0 = \sum_j^m \overset{*}{q}_{j0} / \mathrm{m},$$

Stone derives another formula for an annual base-weighted quantity index, viz.,

$$\overline{q}^*_1 \overline{p}^*_0 / \overline{q}^*_0 \overline{p}^*_0 = \sum_j^m q^*_{j1} / \sum_j^m q^*_{j0}.$$

He accepts this index, in effect an unweighted quantity ratio, as adequate, too, on the ground that "in adjusted units the quantities of different seasons are directly comparable."[59] Accordingly, the use of these units is seen by Stone as also permitting comparisons involving individual seasons; for example, a quantity index for the j-th

[57] R. Stone, op. cit., p. 117.
[58] Ibid., p. 75.
See footnote 59 on p. 274.

season of the current ("1st") year on the base of year "0" as a whole would read

$$q_{j1}^{\bullet}\overline{p}_0/\overline{q}_0^{\bullet}\overline{p}_0^{\bullet}=q_{j1}^{\bullet}/\overline{q}_0^{\bullet}.$$

Price indexes analogous to Stone's quantity indexes are easily identified. The annual Laspeyres formula applied to a single seasonal commodity gives

$$\sum_{j}^{m}p_{j1}^{\bullet}q_{j0}^{\bullet}/\sum_{j}^{m}p_{j0}^{\bullet}q_{j0}^{\bullet}=\sum_{j}^{m}p_{j1}q_{j0}/\sum_{j}^{m}p_{j0}q_{j0}.$$

Given our previous definition of the weighted averages

$$\overline{p}_i^{\bullet}\,(i=0,1\ldots),$$

the result would be identical had the formula

$$\overline{p_1^{\bullet}}\overline{q}_0^{\bullet}/\overline{p}_0^{\bullet}\overline{q}_0^{\bullet}\,(=\overline{p}_1\overline{q}_0/\overline{p}_0\overline{q}_0)$$

been used instead. For the current-season-to-base-year comparison, the corresponding expression is simply

$$p_{j1}^{\bullet}\overline{q}_0^{\bullet}/\overline{p}_0^{\bullet}\overline{q}_0^{\bullet}=p_{j1}^{\bullet}/\overline{p}_0^{\bullet}.$$[60]

To see how Stone's single item formulae can be applied to groups of commodities, let us write out the season-to-year price index for n items $(t=1, 2\ \ldots\ n)$, omitting for simplicity the t-subscripts which would have to be attached to all the p's and q's. The index, which will be recognized as a weighted average of the $p_{j1}^{\bullet}/\overline{p}_0{}^*$ terms, reads

$$\frac{\sum_{t}^{n}p_{j1}^{\bullet}\overline{q}_0^{\bullet}}{\sum_{t}^{n}\overline{p}_0^{\bullet}\overline{q}_0^{\bullet}}=\frac{\sum_{t}^{n}\frac{p_{j1}^{\bullet}}{\overline{p}^{*}}\overline{p}_0^{\bullet}\overline{q}_0^{\bullet}}{\sum_{t}^{n}\overline{p}_0^{\bullet}\overline{q}_0^{\bullet}}=\frac{\sum_{t}^{n}\frac{p_{j1}^{\bullet}}{\overline{p}_0^{\bullet}}\overline{p}_0\overline{q}_0}{\sum_{t}^{n}\overline{p}_0\overline{q}_0}.$$

It would be possible to use various formulae within this framework, for example, to substitute the seasonal for the mean annual q's in the weights. The formulae thus obtained would resemble the seasonal-

[59] Ibid., p. 76. It may be noted that substitution of the weighted average

$$\sum_{j}q_{j0}^{\bullet}p_{j0}^{\bullet}/\sum_{j}p_{j0}^{\bullet}$$

for the unweighted one

$$\left(\sum_{j}q_{j0}^{\bullet}/m\right)\text{ gives }\overline{q_1^{\bullet}}\,\overline{p_0^{\bullet}}/\overline{q_0^{\bullet}}\,\overline{p_0^{\bullet}}=\sum_{j}q_{j1}^{\bullet}p_{j0}^{\bullet}/\sum q_{j0}^{\bullet}p_{j0}^{\bullet}=\sum_{j}q_{j1}p_{j0}/\sum_{j}q_{j0}p_{j0}.$$

This result, obtained by using annual mean figures, is identical with the result obtained in the previous paragraph by using the detailed price and quantity information for each season.

[60] As noted by Stone (op. cit., p. 76), these indexes satisfy the Fisher factor "reversal" test: the product of the matching formulae, $p_j{}^*{}_1/\overline{p}_0{}^*$ times $q_j{}^*{}_1/\overline{q}_0{}^*$, equals the ratio of the current season's value to the base year value. This, however, is trivial in the present case of unweighted price and quantity ratios restricted to the seasonal varieties of a single good.

weight indexes discussed previously in Section II, 5 of this study, and in several cases would actually be equivalent to them. Thus note that

$$\frac{\sum_{t}^{n} p_{j1}^{*} q_{j0}^{*}}{\sum_{t}^{n} p_{j0}^{*} q_{j0}^{*}} = \frac{\sum_{t}^{n} p_{j1} q_{j0}}{\sum_{t}^{n} p_{j0} q_{j0}} \quad \text{and} \quad \frac{\sum_{t}^{n} p_{j1}^{*} q_{j0}^{*}}{\frac{1}{12} \sum_{j}^{m} \sum_{t}^{n} p_{j0}^{*} q_{j0}^{*}} = \frac{\sum_{t}^{n} p_{j1} q_{j0}}{\frac{1}{12} \sum_{j}^{m} \sum_{t}^{n} p_{j0} q_{j0}}.$$

The first of these equations relates to simple same-month-year-ago or same-month-of-the-base-year comparisons (see Section 5a). The second relates to the seasonal index now used in Canada (see Section 5a). In these formulae, then, unadjusted prices and quantities can be replaced by the corresponding adjusted figures without affecting the results. On the other hand, conversion from unadjusted into adjusted units would not leave unchanged Rothwell's formula

$$\sum p_{j1} q_{j0} / \sum \bar{p}_0 q_{j0}$$

(see Section 5d), since the denominator of this index would not in general equal

$$\sum \bar{p}_0^{*} q_{j0}^{*}.$$

Stone illustrates his argument in favor of measurements in the adjusted unit by comparing the simple quantity indexes

$$\sum_{j}^{m} q_{j1}^{*} / \sum_{j}^{m} q_{j0}^{*} \text{ and } \sum_{j}^{m} q_{j1} / \sum_{j}^{m} q_{j0}.$$[61]

He assumes that in the base period the commodity in question was available in large quantities and at nonexorbitant prices only during a small part of the year, while in the current year the progress in refrigeration, development of alternative supply sources, etc., eliminated the wide seasonal differences in the supply of the product, making the latter available throughout the year at more or less similar prices. The adjusted quantity ratio will be higher than the unadjusted one, reflecting the fact that the out-of-season varieties of the commodity, which were highly valued in the base year, are now available in larger amounts.

This is an important argument supported by a realistic example but it is not sufficient to resolve some serious doubts about this approach to the problem of seasonal commodities. The assumption of a negative correlation between the seasonal changes in price and quantity has repeatedly been made on these pages and is no doubt valid for many products (see, however, Part I, Section 3d above). But here, unlike in the other cases, it is built into the method by the device of the inverse relation between the seasonal adjustment factors for prices and quantities. It is possible to question this approach on the ground

[61] R. Stone, op. cit., p. 77.

that it in effect prejudges an issue which had better be left open in the assumptions stage of the analysis.

A secondary practical consideration is that the Stone method would presumably require separate seasonal adjustments for each component item of the index. This is a large although by no means overwhelming task in the case of a comprehensive price index, but the main difficulty here would likely be qualitative rather than quantitative: substantial shifts in the seasonal patterns of some series in the base period and the neighboring years, and the like.[62]

8. SOME RELEVANT ASPECTS OF THE SAMPLING PROBLEM IN INDEX SERIES CONSTRUCTION [63]

In a recent article on the probability sampling approach to the making of price indexes, the claim is made that this method, in contrast to the present "use of an arbitrary fixed sample," would "permit changes in products and product quality to be incorporated smoothly into a continuing index." [64] This apparently implies that the suggested sampling procedure will result in an index series for which seasonal changes (along with such other important factors as the nonseasonal weight and quality changes) would not present any major difficulty in principle.

This claim, if so interpreted, is believed to be excessive and potentially misleading. The matter deserves some attention, although it is difficult to discuss it without digressing somewhat from our main line of discourse. But first it must be emphasized that what follows is not at all intended to question the objective sampling per se or its advantages over the currently used judgment sampling.[65] The application of probability sampling to price index construction is an important task to which Adelman [66] and, before her, Banerjee [67] have made valuable contributions.

The proper use of sampling in this connection is within strata or "composite commodities," i.e., groups of items with common patterns

[62] Stone himself devotes most of his chapter on "Seasonal Variations" (op. cit., pp. 77–88), not to the treatment of seasonal commodities, but rather to the task of developing a satisfactory method of seasonal adjustments. His basic treatment of the subject as a problem in the analysis of variance is admirable, as are the further refinements of his analysis.

[63] The author is grateful to Professor Philip J. McCarthy of Cornell University for a valuable criticism of an earlier version of this section and suggestions that helped to improve it.

[64] Irma Adelman, "A New Approach to the Construction of Index Numbers," *The Review of Economics and Statistics*, Vol. XL, No. 3, August 1958, pp. 240–249 (the quotations in the text are from pp. 240 and 247, respectively).

[65] On the contrary, these advantages are seen as very substantial. If worked out satisfactorily, the objective sampling procedures would provide estimates of standard errors, which are not presently available for our major price indexes, and thus also the possibility of improving the sampling precision of these indexes.

[66] Adelman, op. cit.

[67] See "A Comment on the Sampling Aspects in the Construction of Index Numbers," *The Review of Economics and Statistics*, Vol. XLII, No. 2, May 1960, pp. 217–218, and the list of the pertinent writings by K. S. Banerjee, ibid., footnote 2.

of price change (relative to the general price level movements which are taken to affect all such groups). It is not among the strata whose relative prices follow distinctly different courses over time.[68] Hence the appropriate form of sampling presupposes a satisfactory solution of an important and difficult task—the grouping of the index items into strata. The components of a stratum must meet the criterion of a reasonably close similarity of relative price change, so that they would presumably belong to a rather narrow cluster of good substitutes produced under generally similar supply conditions. One would hope that a stratification based on this criterion would not have to be revised too often over time, but the degree of stability achievable in this respect for an economy as dynamic as that of the United States might prove considerably less than the practical index maker would wish.

In the absence of any changes in the availability or quality of the goods that make up the universe to be covered by the price index, the probability sampling approach as proposed by Banerjee and Adelman would face no major theoretical difficulties. If we assume a stable division of the universe into a (presumably large) number of proper strata, an intrastratum sampling scheme consistent with the currently dominant fixed-weight type of index numbers could be adopted. In-

[68] Implied in the statistical sampling procedures is the assumption that the price change of each item can be decomposed into three independent additive parts: (1) that common to all items in the universe; (2) that common only to the items within the relevant strata; and (3) a random component. The weighted average of the elements (2) is zero for the economy as a whole, the weighted average of the elements (3) is zero for each stratum. (See Adelman, "Reply," *The Review of Economics and Statistics*, May 1960, p. 219.) As a working arrangement this is presumably acceptable, even though the real world, in which prices are interdependent and changes in their structure may affect their level (as well as vice versa), is undoubtedly very different from the above model of independent price changes (1) and (2). But the construction seems to us just strong enough to permit sampling within carefully selected groups of related items; it will not bear either sampling of the items directly in disregard of such strata or sampling among the strata.

Ideas which seem to suggest sampling beyond the range of the "composite commodities" go back to Edgeworth (1887) and are shared also in similar form by W. S. Jevons (1909) and Bowley (1928). This is the conception that any change in the general price level or in the value of money "in itself" should manifest itself in a proportional change of all prices. Monetary factors are supposed to act upon each price alike and deviations from proportionality are viewed as due to other causes; but if this is so, then such deviations can be treated as if they were errors of observations as far as the measurement of price level changes is concerned. If a sufficiently large number of observations of any individual prices is taken, their relative movements will cancel out in accordance with the law of error and the residual movement of the price level will be satisfactorily measured by the average, subject to the ordinary sampling errors, etc. The logic of this approach does not call for weighting of the price relatives sampled according to the economic importance of the goods concerned. Rather, if weights are applied they should vary with the degree of precision of the individual observations.

The principal objection to this "stochastic approach" (Frisch) is that it implies that individual prices show divergencies from the "true" average price level that are independent from each other and that their fluctuations around that (moving) level are of a random character. Monetary as well as nonmonetary factors may exert different amounts of influence on prices of different goods. "Economic" weighting of the index items is essential to impart to the price level concept a definite meaning. Extensive criticism of the "stochastic" approach along the above lines is found in J. M. Keynes, *A Treatise on Money*, Vol. I, pp. 79–88 (1st ed., 1930). Similar arguments have also been made by Welsh (1924), Gini (1924), Divisia (1925), and Frisch (1936; see his article in footnote 9 for references).

deed, schemes of this sort are provided in the Banerjee-Adelman proposals.[69]

In reality, commodity universes change continually over time, and, here as elsewhere, it is this change that creates the major problems. Variation in the assortment of goods available to or desired by the buyers will in the course of time invalidate even the most carefully implemented, detailed stratification schemes. Some of this variation can be predicted to a considerable extent, the stable elements in the seasonal change being here of particular importance. Such changes should be taken into account in the stratification design as extensively as possible, in order to make that design better and more durable. But that part of the variation which is nonrecurrent and largely unpredictable—most of the changes in quality and many of those in weights—cannot be given such an advance treatment with fair chances of success.

Apart from the stratification problem (or assuming, boldly, an enduring satisfactory solution to it), the question arises as to how frequently new samples of products should be drawn and priced in the process of producing the index series. Strict sampling considerations suggest drawing a completely new sample of items for each pricing period (month, in the major U.S. indexes), but other reasons militate against this extreme course. The operation would presumably be very costly. Moreover, it would be necessary to chain the resulting monthly links into a continuous series, a procedure which, as we know, gives rise to errors of its own.[70] To have a practical chance of being adopted and proving workable, the probability sampling method would probably have to be considerably attenuated to permit some compromise with the constant-market basket (fixed-commodity sample) principle of the price index maker.

9. BASIC CONCLUSIONS AND SUGGESTIONS

None of the various approaches that we have systematically explored is free from deficiencies, but some of these are much more

[69] Adelman suggests that each of the n items in a sample from a given stratum be assigned a probability of selection which is proportional to its weight (w_i) in the stratum; then a simple (unweighted) mean of the selected price relatives (p_i), that is

$$\frac{1}{n}\sum_{i=1}^{n} p_i,$$

will provide an unbiased estimate of the weighted average of the price relative for the entire stratum,

$$\sum_{i=1}^{N} p_i w_i$$

(where N=total number of items in the stratum). To meet the consistency requirement posed in the text, let w_i be the normalized base year expenditure weights.

It may be noted, however, that with respect to weights Adelman's position shows some affinity with the "stochastic approach," a critical summary of which was given in footnote 68. Although Adelman did use expenditure weighting in her pilot study index, she feels that this was an "arbitrary" assignment and that "just about any a priori weighting scheme would permit a reasonable evaluation of the whole procedure." Again, while following the procedure of making the number of items drawn from each stratum roughly proportional to the stratum's (expenditure) weight, she observes that this is a mere expedient: optimally, "the sampling percentage in each group ought to vary directly with the standard deviation of that group" (Adelman, op. cit., pp. 244–245). But then it should also be mentioned that these points are probably of rather marginal concern to Adelman who is well aware that her proposal "would not solve the problem of appropriate weighting" (p. 240).

[70] See Adelman (op. cit., p. 244) for an interesting variance ratio formula derived by R. Dorfman which shows that sampling errors will tend to be larger for the chained than for the fixed-base index.

serious than others. Thus it is possible to eliminate certain methods and discriminate among the remaining ones.

Two objectives may be distinguished, one limited and one comprehensive. The first is that of identifying a preferable method of dealing with "part-year" items or the effective seasonal disappearances (technically the most troublesome aspect of the seasonal problem). The second is that of finding the most satisfactory way of coping with the seasonal problem as a whole, including the issue of seasonally disappearing or unique goods.

(1) The procedure of imputing out-of-season items to their groups yields results whose quality will depend upon what precisely is imputed to what; a general unconditional prediction of how this method will work is not possible. The method can be viewed as an intragroup expenditure-weight transfer or a substitution of "year-round" or in-season commodities for "part-year" or "out-of-season" commodities. Seasonal substitutions are real and often important phenomena, but their incidence does not necessarily conform to the groupings adopted in subclassifying the price index in question. A group imputation may therefore ignore or cut across the real seasonal substitution relationships. If so, it may give poor or even perverse results, which could conceivably be inferior to those obtained by the other method applied to the problem of seasonal disapperances—the practice of holding the prices of out-of-season items constant. However, errors of application aside, the method of seasonal substitution is the logically preferable of the two, as it enables an annual weight index to give some—very limited but pertinent and opportune—recognition to the seasonal variation in consumption. But the proper application of this method presupposes a comprehensive and detailed study of the substitution relations involving seasonal commodities.

(2) If seasonal (say, monthly) quantities are used in weights, instead of annual quantities, two basic approaches are available. One is to use a standard (base year) set of seasonal weights over a number of years, as long as the set seems sufficiently realistic. The other is to use varying sets of seasonal weights from year to year to meet the changing exigencies of the particular year. This is the familiar dichotomy between the "fixed-base" and the "chain" indexes applied to the seasonal-weight measures.

Where seasonal fluctuations are pronounced and relatively stable, chain indexes, which fail to meet the proportionality test, will tend to produce errors in the year-to-year (same season) comparisons. Over a period of years, chain methods could not be properly applied without corrections for the "drifts" which would then tend to develop. Under these conditions, the use of a base year set of seasonal weights is preferable.[71] But, again, we know that this approach faces the critical difficulty with the month-to-month comparisons, and its more or less conventional variants (surveyed in Section II, 5 above) all fail in one way or another to provide an adequate solution to the problem.

Suppose, however, that one succeeded in ascertaining seasonal market baskets of such a composition as would give the "average consumer" approximately equal utility or satisfaction in each month of

[71] In terms of practicability and cost, this "fixed-base" approach has, of course, always a big advantage over the chain methods which presuppose a continuous collection of current seasonal weight data.

the year. (The Bortkiewicz formulae and the Staehle method of analyzing differential consumption structures, which have been reviewed in Section I, 3c above, might be profitably used in this connection.) Provided that—and as long as—the seasonal pattern of consumption remains sufficiently stable, such a set of market baskets would have to be selected only for the base year.[72] An index of this type would solve the basic difficulty of month-to-month measurements in the seasonal context; the same-month-year-ago comparisons, for which a constant market basket is assumed, have of course presented no problem to begin with and retain their conventional form.

Given the proper monthly market baskets, the simplest method of seasonal index construction can then be used, viz, comparisons of prices for a given month with those for the same month of the base year, using quantities for the latter period in weights. There would be no need to employ any of the more complicated formulae used or proposed as solutions to the seasonal problem; indeed, each of these formulae has one or another disadvantage which argues against its acceptance.

To be sure, this approach requires some departure from the strict concept of a price index in the direction of a cost-of-living index; but then some relaxation of the former concept will always be necessary if one wants to really come to grips with the seasonal problem. It is also clear that the empirical application of the method would be a task of major proportions and probably of considerable difficulty. But good results (even if obtainable only for some portions of an index which show large and sufficiently stable seasonalities) would here presumably pay off considerable investment in data collection and research.

III. Some Statistical Explorations of Seasonalities in Quantities and Prices

Knowledge of seasonal changes in each of the individual price series used in the computation of a comprehensive price index is not in itself sufficient to provide knowledge of the "true" seasonal variation in that index as a whole. This, of course, is an implication of the "seasonal weight problem" that has been given much attention in the present study. Nevertheless, measures of seasonal movements in prices of individual commodities or product classes are clearly of great interest, assuming their quality is adequate. When available for a large number of items, including those of major relative importance, such measures convey valuable insight into the dimensions of seasonal influences upon the movements of prices. How large is the proportion of the price series that show substantial seasonal fluctuations? How large and persistent are these fluctuations? What is the degree of confluence of the seasonal patterns among the various price series? These questions, whose pertinence will hardly be doubted, lend themselves to an empirical investigation, and the recently computed seasonal adjustment factors for the BLS price index series provide data that promise some progress in this direction.

[72] But the method is, of course, perfectly compatible with basic weight revisions every few years or so.

TABLE V.—*Range of Average Seasonal Indexes for Selected Groups, Subgroups, Product Classes, and Items of the Consumer Price Index, 1947–58*

Line	Group or item	Range of average seasonal index [1] (1)	Rank [2] (2)
	A. GROUPS AND SUBGROUPS [3]		
1	All items	0.7	34.5
2	All items, less food	.6	36
3	All commodities [4]	1.1	30
4	All commodities less food [4]	1.1	30
5	Durable commodities [4]	1.5	23.5
6	Nondurable commodities less food [4]	1.0	33
7	Food	2.4	19
8	Food at home	2.7	17
9	Meats, poultry, and fish	6.1	8
10	Meats	7.5	7
11	Dairy products	3.4	14
12	Fresh fruits and vegetables	15.3	5
13	Apparel	1.2	26.5
14	Housefurnishings	1.1	30
15	Appliances [5]	1.3	25
16	Private transportation	1.5	23.5
	B. PRODUCT CLASSES AND ITEMS [6] [7]		
17	Tomatoes [8]	60.3	1
18	Potatoes	25.5	2
19	Oranges	24.2	3
20	Eggs	23.1	4
21	Pork	12.4	6
22	Poultry	5.8	9
23	Beef and veal	5.2	10
24	New cars [9]	4.8	11
25	Milk, fresh (grocery)	4.7	12
26	Used cars [8]	4.5	13
27	Solid fuel—fuel oil	3.3	15
28	Bituminous coal	3.2	16
29	Fish	2.6	18
30	Fats and oils	2.3	20
31	Women's and girls' apparel	2.2	21
32	Refrigerators, electric [5]	1.6	22
33	Television [10]	1.2	26.5
34	Textile housefurnishings [10]	1.1	30
35	Gasoline [11]	1.1	30
36	Men's and boys' apparel	.7	34.5
37	Footwear	.5	37
38	Furniture [5]	.4	38

[1] Derived from average monthly seasonal indexes for 1947–58, except when a footnote to the contrary is attached to the title of the series.
[2] Based on the entries in col. 1, from the largest (rank 1) to the smallest (rank 38).
[3] Includes overall aggregates and groups containing any of the items listed in Part B below.
[4] Based on quarterly data, 1947–55; monthly data, 1956–58.
[5] Based on the average quarterly seasonal index, 1947–58.
[6] Listed according to their ranks in col. (2).
[7] Includes some groups for whose components no separate price seasonals are available (see note 3).
[8] Based on the average monthly seasonal index, 1953–58.
[9] Based on quarterly data, 1947–52; monthly data, 1953–58.
[10] Based on the average quarterly seasonal index, 1951–58.
[11] Based on quarterly data, 1947–56; monthly data, 1957–58.

1. SEASONAL MOVEMENTS IN PRICES OF CONSUMER GOODS

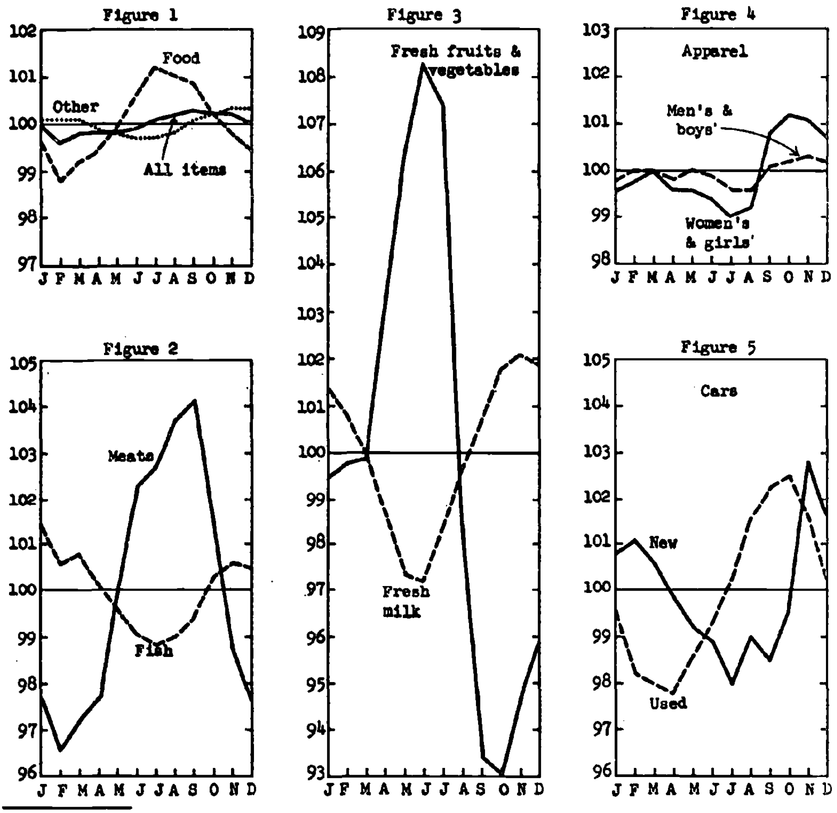
The most outstanding single feature of these movements is their extraordinary diversification. The amplitudes of the average seasonal indexes for a sizable sample of the U.S. Consumer Price Index (CPI) series covering the period 1947–58 range from about 60 to less than 0.5 percent (Table V). Fresh fruits and vegetables and then eggs lead the list with amplitudes exceeding 20 percent of the corresponding average annual levels. In the 5 to 12 percent range there are meats; in the 3 to 5 percent range, milk, new and used cars, and fuels. The remaining ten items (out of the 22 listed in Part B of

Table V) have amplitudes of less than 3 percent. The figure for women's and girls' apparel, for example, is only 2.2 percent.[78]

The average seasonal indexes for the CPI components vary not only in their amplitudes but also in their patterns or the timing, within the year, of their upward and downward movements. Some prices rise seasonally early and decline late in the year, others behave in the opposite fashion. Chart 1 gives several illustrations: prices of meats increase seasonally from February to September, those of fish from June to November (Chart 1, Fig. 2); prices of fresh fruits and vegetables rise from October to June, those of fresh milk from

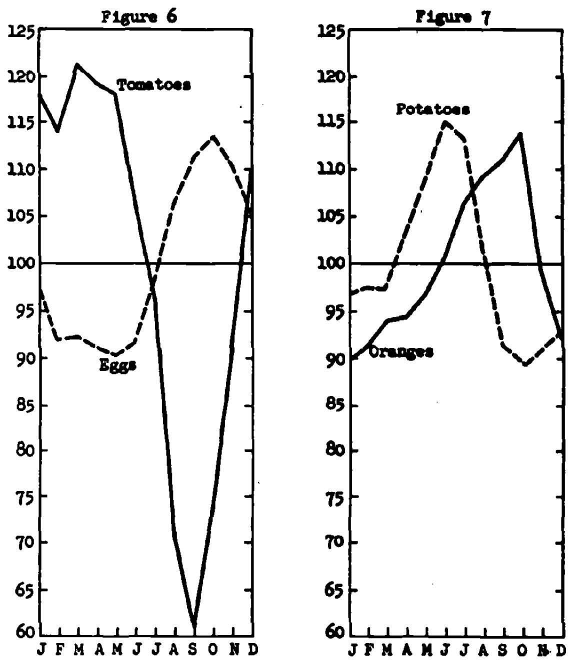
CHART 1

Average Seasonal Indexes for Selected Groups and Items of the Consumer Price Index



[78] It should be noted, however, that these measures relate to certain product classes; the seasonals for specific items within these categories would often show larger fluctuations. Also, each of these measures is based on a set of averages of the seasonal factors for Januaries, Februaries, etc., of all the years covered by the seasonal index in question (the data used in Table V all refer to the 12-year period beginning in 1947, except for two shorter series). This averaging over time is likely to produce amplitudes that are somewhat dampened in comparison with the amplitudes of the indexes for the individual years. This effect, however, should as a rule be weak because the averaged figures are "moving" seasonal factors which vary but slightly from year to year, reflecting the notion that, typically, seasonal movements are fairly stable and changes in them are mostly gradual. (The seasonal factors were all uniformly obtained by the same method of seasonal adjustment: the electronic computer program of the Bureau of the Census based on a rather elaborate version of the ratio-to-the-moving-average approach. For a description of this program, see Julius Shiskin, *Electronic Computers and Business Indicators*, Occasional Paper 57, National Bureau of Economic Research, New York, 1957.)

CHART 1—Concluded



Source: U.S. Department of Labor, Bureau of Labor Statistics.

June to November (Chart 1, Fig. 3). The tomatoes-eggs contrast (Chart 1, Fig. 6) is particularly sharp. To be sure, these examples of almost inverse patterns are somewhat on the extreme side. In many instances, comparison of the seasonals reveals shorter timing differences and a larger number of months with the same direction of movement. Thus the divergent seasonal movements of new and used car prices are concentrated mainly in the April–July interval (Chart 1, Fig. 5). Small amplitude differences alone distinguish the price seasonals for the two categories of apparel in Fig. 4 (Chart 1) as timing differences between them are very slight.

Because the seasonal movements of its component price series offset each other to a large extent, average seasonal changes in the CPI as a whole, in its present form, are of a very small order of magnitude (Chart 1, Fig. 1). The amplitude of the average seasonal pattern for the major group of foods is much larger, but still small compared with that of any of the patterns for the individual food items covered by our measures (Table V). The other major groups combined have an average seasonal index that moves in the opposite direction to the food index in most months but is much flatter. As shown in Chart 1, Fig. 1, the overall index for the CPI resembles more the food seasonal in the direction, and more the "other items" (nonfood) seasonal in the size, of movements. (In terms of value weights developed from the 1950 consumer expenditures survey adjusted to December 1952 prices, foods accounted for 30 percent of the total CPI.)

While the monthly change in the seasonal index for all items of the CPI combined was on the average only ±0.13 percent in 1947–58, it is nevertheless true, as observed in the introduction to this study, that "seasonal influences may and at certain times did dominate the short-run behavior of . . . the Consumer Price Index." It should be noted that the total CPI is a sluggish series; under more or less "normal" peacetime conditions, e.g., during most of the recent post-Korean period, the index would not often vary from month to month by more than 0.1 or 0.2 of an index point (which in percentage terms amounts to still smaller changes). Moreover, among the components of the CPI that are particularly stable in the short run, those that show little seasonal sensitivity appear to have the greatest importance. Thus changes in prices of the seasonal items will frequently be allowed to exert a relatively strong influence upon the month-to-month behavior of the total index.

2. SEASONAL MOVEMENTS IN PRIMARY-MARKET PRICES

The evidence on seasonal fluctuations in the components of the U.S. Wholesale Price Index (WPI) is presented in this section in the same way as was the evidence for the CPI items in the preceding section. This will save description space and facilitate comparisons. Again, the dominating impression is that of diversity. Among the average seasonal amplitudes for prices paid in the primary markets, a number exceed the largest of such amplitudes for prices paid by consumers, so that the range of the former measures is still considerably wider than that of the latter (cf. Parts B of Table V and VI). In these terms, then, wholesale prices are found to be on the whole more sensitive seasonally than the consumer price indexes.[74] A comparison of the measures for the comprehensive series (in Parts A of the two tables) also provides some evidence of the same relation, although these amplitudes are small for both the CPI and the WPI for the already familiar reason, the offsetting seasonals in the component price indexes.

Again, several kinds of vegetables and fruits lead the list with the largest seasonal amplitudes—in excess of 30 percent for eight items. Meats, poultry, livestock, hides, eggs, and milk are found in the middle range. Other items—about half of the total collection—show amplitudes of less than 5 percent. They include predominantly durables, both producer and consumer goods, but also fuels and apparel (see the rankings in Table VI).

Chart 2 parallels to a certain extent Chart 1 and shows that the seasonals for the WPI items, too, vary greatly in their patterns. For example, the index for fresh fruits contrasts sharply with that for fluid milk (Chart 2, Fig. 3). Florida and California oranges have quite different seasonals (Chart 2, Fig. 10). Prices of two types of lumber show almost entirely inverse (but small) seasonal fluctuations (Chart 2, Fig. 7). Other diagrams (e.g., Chart 2, Figs. 2 and 6) illustrate smaller timing differences and partial overlaps. Some comparisons show relationships that are very similar to those found for the corresponding consumer price series (cf. Figs. 8 and 10 in Chart 2 with Figs. 6 and 7 in Chart 1).

The indexes for "farm products" and "all commodities less farm and food" (Chart 2, Fig. 1) are broadly similar in direction of move-

[74] It will be recalled that over the cycle, too, wholesale prices have historically tended to fluctuate more widely than retail prices.

TABLE VI.—*Range of Average Seasonal Indexes for Selected Groups, Subgroups, Product Classes, and Items of the Wholesale Price Index, 1947–58*

Line	Group or item	Range of average seasonal index [1] (1)	Rank [2] (2)
	A. GROUPS AND SUBGROUPS [3]		
1	All commodities	0.8	41.5
2	All commodities less farm and food	1.1	36
3	Farm products	2.8	28
4	Fresh fruits	14.0	17
5	Fresh and dried vegetables	31.0	9
6	Processed foods	2.1	30
7	Meats	8.9	22
8	Textile products and apparel	1.2	32.5
9	Hides and skins	9.1	21
10	Lumber and wood products	0.7	44
11	Lumber	0.8	41.5
	B. PRODUCT CLASSES AND ITEMS [4] [5]		
12	Snap beans	93.8	1
13	Cabbage	72.0	2
14	Tomatoes	63.4	3
15	Oranges, Florida	60.1	4
16	Carrots	41.2	5
17	Potatoes, white, Chicago	40.6	6
18	Onion	38.2	7
19	Lettuce	34.2	8
20	Pork loins, fresh	29.2	10
21	Celery [6]	26.0	11
22	Barrows and gilts, 200–240 pounds	22.0	12
23	Oranges, California	18.9	13
24	Lemons	17.0	14
25	Cattle hides	16.4	15
26	Live poultry	15.7	16
27	Eggs	11.0	18
28	Beef, choice	9.9	19
29	Livestock	9.8	20
30	Fluid milk	7.9	23
31	Steers, choice	5.0	24
32	Coal	3.5	25
33	Douglas fir lumber	3.2	26
34	Leather	3.1	27
35	Southern pine lumber	2.3	29
36	Agricultural machinery	1.3	31
37	Gasoline	1.2	32.5
38	Construction machinery	1.1	36
39	Household furniture	1.1	36
40	Commercial furniture	1.1	36
41	Structural clay products	1.1	36
42	Apparel	1.0	39
43	House appliances	0.8	41.5
44	Concrete ingredients	0.8	41.5

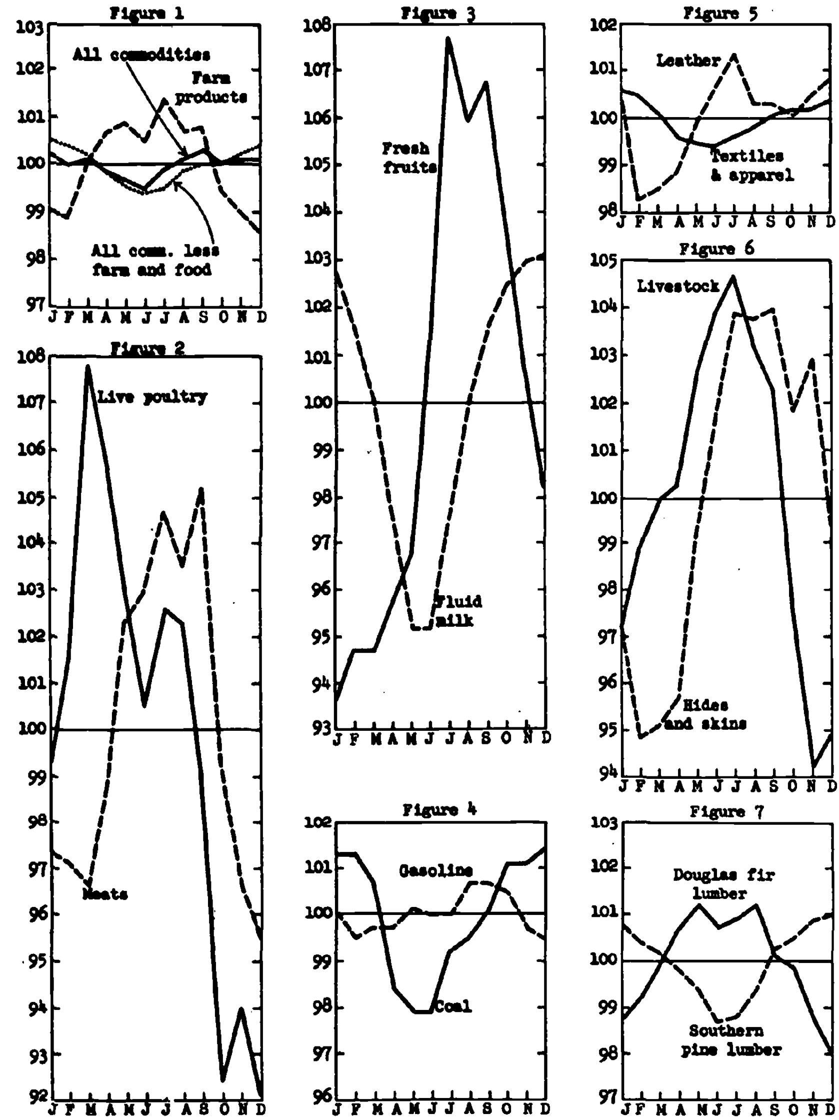
[1] Based on average monthly seasonal indexes for 1947–58 (except line 21).
[2] Based on entries in column 1, from the largest (rank 1) to the smallest (rank 44).
[3] Includes overall aggregates and groups containing any of the items listed in Part B below.
[4] Listed according to their ranks in column 2.
[5] Includes some groups for whose components no separate price seasonals are available (see note 3).
[6] Based on the average monthly seasonal index, 1950–58.

ment to the indexes for the CPI groups "food" and "all items less food" (Chart 1, Fig. 1), but are somewhat larger in amplitude. The seasonal pattern of the total WPI ("all commodities") resembles rather closely that of commodities other than farm products and processed foods. The CPI seasonal, on the other hand, is apparently influenced relatively more by food and less by other items.[75]

[75] The relative importance within the WPI of farm products and processed foods combined is about 30 per cent—much like the relative importance of foods within the CPI.

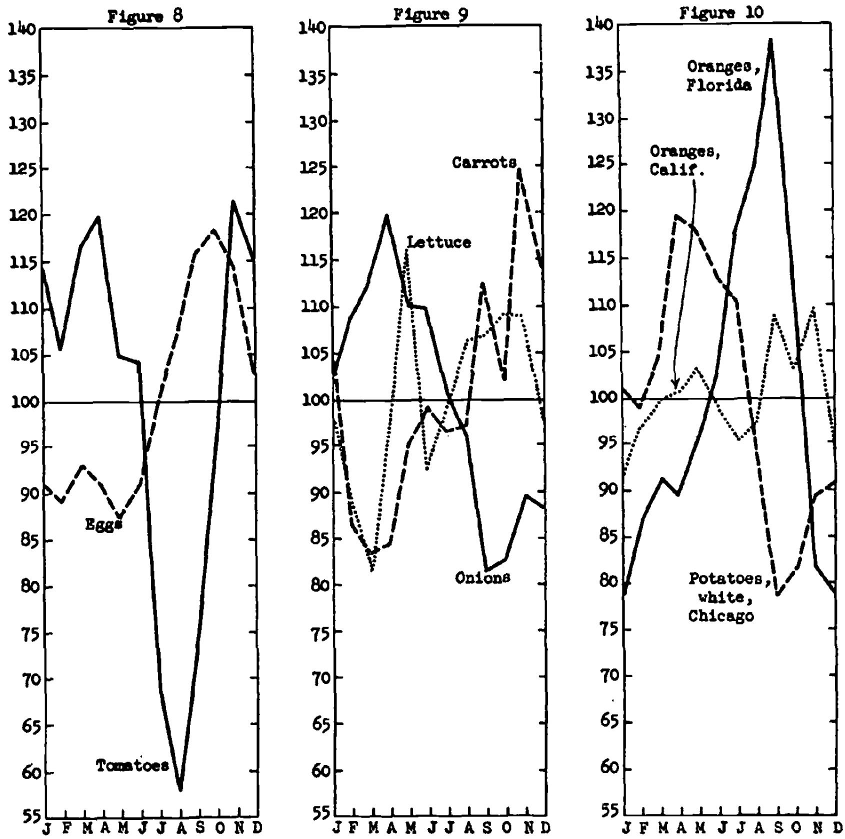
CHART 2

Average Seasonal Indexes for Selected Groups and Items of the Wholesale Price Index



Source: U.S. Department of Labor, Bureau of Labor Statistics.

CHART 2—Concluded



Source: U.S. Department of Labor, Bureau of Labor Statistics.

3. SEASONAL MOVEMENTS IN PRICES RECEIVED BY FARMERS

Many component items of the Index of Prices Received by Farmers fluctuate widely over the seasons. Index numbers of seasonal variation have been computed by the Agricultural Marketing Service, U.S.D.A., and published for all those price series that have significant and not excessively erratic seasonal patterns based on sufficiently long and comparable historic data. The average amplitudes of these indexes are listed in Table VII.

Not surprisingly, the commodities with the largest seasonal amplitudes are here again fresh vegetables and fruits. These, together with potatoes, account for the entire first half of the list (lines and ranks 1–22 in Table VII).[76] Few generalizations can be made about the other commodity groups which include prices with intermediate or small seasonals, but the relatively high ranks of wholesale milk and

[76] Among the items with the very largest amplitudes are some that have short marketing seasons and can be priced directly only in certain months of the year (cf. Table VII, lines 1–3, 7 and 11).

TABLE VII.—*Range of Average Seasonal Indexes for Selected Items of the Index of Prices Received by Farmers*[1]

Line	Commodity[2]	Group	Range of average seasonal index
1	Asparagus[3]	Commercial vegetables for fresh market	142
2	Watermelons[3]	do	141
3	Cantaloupes[3]	do	123
4	Cucumbers	do	109
5	Peppers, green	do	98
6	Corn, sweet	do	85
7	Tangerines[3]	Fruits	84
8	Tomatoes	Commercial vegetables for fresh market	77
9	Grapefruit	Fruits	73
10	Spinach	Commercial vegetables for fresh market	69
11	Strawberries[3]	Fruits	62
12	Onions	Commercial vegetables for fresh market	57
13	Carrots	do	56
14	Beans, snap	do	51
15	Cabbage	do	46
16	Sweet potatoes	Potatoes, etc	39
17	Oranges excluding tangerines	Fruits	37
18	Celery	Commercial vegetables for fresh market	30
19	Lemons	Fruits	29
20	Lettuce	Commercial vegetables for fresh market	28
21	Cauliflower	do	25
22	Potatoes	Potatoes, etc	24
23	Milk, wholesale	Dairy products	22
24	Soybeans	Oil-bearing crops	20
25	Hogs	Meat animals	18
26	Eggs	Poultry and eggs	18
27	Sheep	Meat animals	17
28	Corn	Feed grains and hay	17
29	Turkeys	Poultry and eggs	16
30	Apples	Fruit	16
31	Flaxseed	Oil-bearing crops	16
32	Oats	Feed grains and hay	15
33	Grain sorghums	do	14
34	Broccoli	Commercial vegetables for fresh market	13
35	Cottonseed	Oil-bearing crops	12
36	Rice	Food grains	12
37	Chickens	Poultry and eggs	10
38	Beef cattle	Meat animals	10
39	Lambs	do	10
40	Calves	do	9
41	Hay, baled	Feed grains and hay	9
42	Barley	do	8
43	Rye	Food grains	8
44	Wheat	do	7
45	American upland	Cotton	7

[1] The seasonal indexes are based on ratios to centered 12-month moving averages for the following periods of years: Meat animals (lines 25, 27, and 38–40): 1921–53 (excl. 1942–46); corn, barley, rye, wheat and cotton (lines 28 and 42–45): 1923–52; potatoes, oats, rice, and chickens (lines 22, 32, 36, and 37): 1933–52; turkeys and grain sorghums (lines 29 and 33): 1934–52; hay (line 41): 1925–52; oil-bearing crops (lines 24, 31, and 35): 1947–51; sweet potatoes (line 16): July 1940–June 1954; eggs (line 26): 1954–58; lemons (line 19): 1938–58; apples (line 30): 1944–58; all other fruits, all commercial vegetables, and milk (line 1–18, 20–21, 23, and 34): 1948–58.

[2] Listed according to the seasonal range, from largest to smallest.

[3] Pricing season is less than a year.

Source: U.S. Department of Agriculture, Agricultural Marketing Service Crop Reporting Branch.

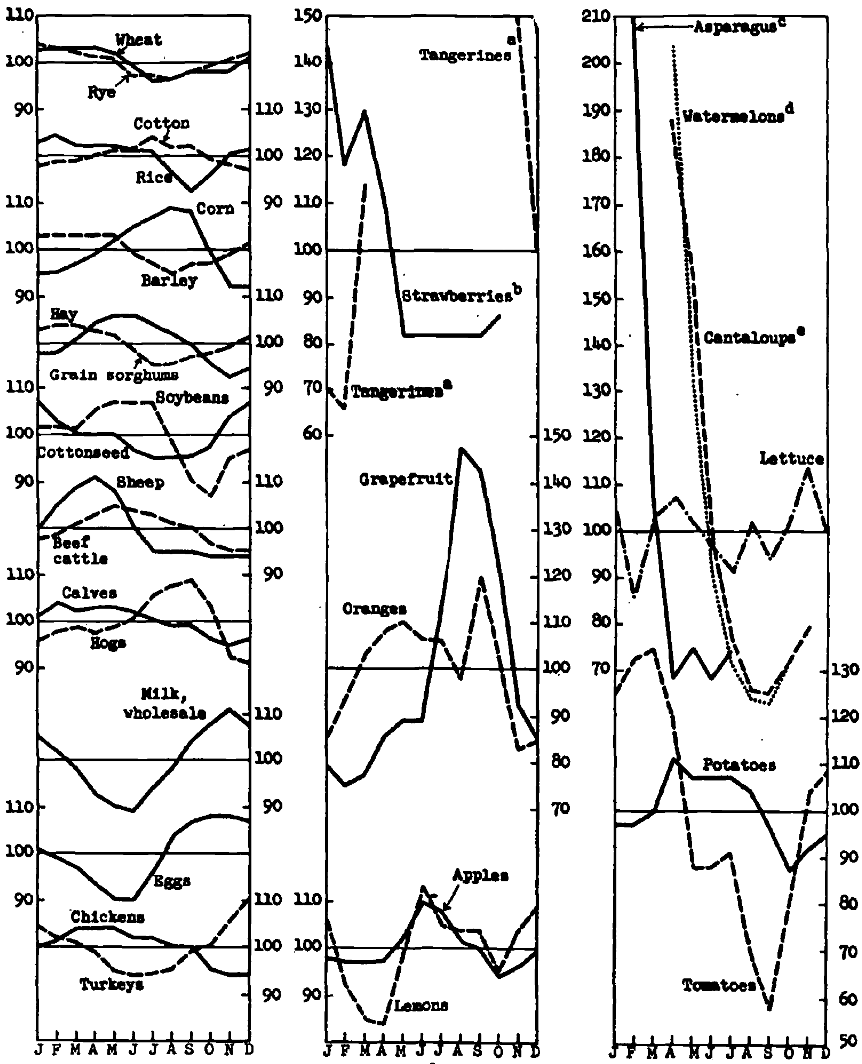
eggs and the position at the bottom of the list of food grains and cotton may be noted.

The large variety of seasonal patterns found among prices received by farmers is demonstrated in Chart 3. Some striking contrasts will be noted there between indexes for items belonging to the same commodity groups. For the All Farm Products Index of Prices Received by Farmers, no index of seasonal variation is computed since the patterns of the various component price series are virtually offsetting. For some commodity groups, however, index numbers of prices received are published both in the seasonally unadjusted and adjusted form.[77]

[77] These groups are: (a) fruit; (b) commercial vegetables for fresh market; (c) potatoes, sweet potatoes, and dry edible beans; (d) dairy products; and (e) poultry and eggs.

CHART 3

Average Seasonal Indexes for Prices Received by Farmers, Selected Commodities



[a] Pricing season: November-March.
[b] Pricing season: January-October.
[c] Pricing season: February-July.
[d] Pricing season: April-October.
[e] Pricing season: April-November.

Source: Crop Reporting Board, Agricultural Marketing Service, USDA.

4. SEASONAL MOVEMENTS IN QUANTITIES

Data on short-run changes in quantities consumed, and in particular on their seasonal variation, are very scanty. For large groups of products and on a national basis, this information is not available at all at the present time. In fact, lack of quantity data of adequate coverage and in suitable form is a major stumbling block that would have to be laboriously removed should an attempt be made to use seasonal weights in the construction of the principal U.S. price indexes.

Foods is the only major commodity group for which a large amount of material on seasonal variation in quantities consumed is available. The U.S. Bureau of Labor Statistics prepared a detailed tabulation on the "Estimated Relative Change in Quantities of Selected Foods Purchased per Month" and kindly gave us permission to make restricted use of these materials for the purpose of this study. The Bureau describes these data as "derived from various sources and selected as appropriate for use with average weekly expenditures for food items reported by households in Chicago, Ill., in Spring 1951." Table VIII presents a summary of these data by what is regarded as their single most significant characteristic, namely the size of the

TABLE VIII.—*Indexes of Seasonal Change in Quantities of Food Items Purchased per Month, Distribution by Group and Amplitude Range*

Line	Group	Number of items [1]	Indexes of seasonal change in quantities purchased						Number of items with no seasonal
			Total number	Number within specified amplitude range					
				Less than 50 percent	50 to 99 percent	100 to 149 percent	150 to 199 percent	200 to 250 percent	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	Milk, cream, ice cream, cheese	11	5	5	-----	-----	-----	-----	2
2	Fats and oils	12	9	9	-----	-----	-----	-----	-----
3	Eggs, meat, poultry, fish	51	19	15	4	-----	-----	-----	-----
4	Potatoes, peas, beans, and nuts	12	9	2	3	1	3	-----	-----
5	Fresh fruits	8	6	1	1	2	-----	2	1
6	Fresh vegetables	15	14	2	2	2	5	3	1
7	Canned, frozen, and dried fruits	19	16	11	4	-----	1	-----	-----
8	Canned, frozen, and dried vegetables	23	12	10	2	-----	-----	-----	-----
9	Sugars and sweets	12	5	4	1	-----	-----	-----	1
10	Grain products	31	16	13	3	-----	-----	-----	4
11	Miscellaneous and unspecified [2]	17	8	7	1	-----	-----	-----	3
12	Total (groups 1-11)	211	119	79	21	5	9	5	12

[1] A number of these items form groups of two or more which have the same seasonal patterns. Hence the total number of items (211) exceeds considerably the total number of the various seasonal indexes estimated (119; see col. 2). A few items, too, have been found to show no significant seasonal variation (see col. 8).

[2] Mainly beverage and accessories, and also baby foods.

Source: Special tabulation made available by courtesy of the U.S. Bureau of Labor Statistics. Derived from various sources and selected as appropriate for use with average weekly expenditures for food items reported by households in Chicago, Illinois, Spring 1951.

seasonal movement. The classification by food categories employed in this table is such that items in different groups do not, while items in the same group often do, have common seasonal patterns.

The table shows that the seasonal indexes for food consumption reach into the ranges of extremely large amplitudes. Nineteen, or about one-sixth, of them show amplitudes in excess of 100 percent; fourteen, or more than one-eighth, exceed 150 percent; and a few even exceed the 200 percent mark (Table VIII, cols. 5–7). In contrast, the four most pronouncedly seasonal of the consumer prices listed in Table V (lines 17–20) show amplitudes of only 23–26 and (in one case) 60 percent, and the four largest seasonal amplitudes for wholesale prices in Table VI (lines 12–15) fall between 60 and 94 percent. While the samples of the price and of the quantity seasonals leave much to be desired in regard to comparability, the above observations refer to prices and quantities of similar if not identical commodities. The comparisons could be extended further with analogous results. Hence the inference seems warranted that seasonal movements tend to be larger in quantities purchased than in prices, at least for many food products.

The commodities with seasonal consumption amplitudes of 100 percent or more all belong to the fruits and vegetables categories (15 items, all but one fresh products) and to the group of potatoes, peas, beans, and nuts (4 items). These highly seasonal commodities account for the bulk of items in these three product groups (Table VIII, lines 4–6). Of the twenty-nine items in these groups, only five have amplitudes of less than 50 percent. As will be recalled from Tables V and VI, the same groups also show the largest seasonal amplitudes of *price* movements. This, of course, is due to the conditions of supply of these commodities, which account for the seasonality of both their consumption and their prices.

Chart 4 illustrates the great diversity of seasonal patterns in quantities purchased of the various food products. Again, as in the diagrams for price seasonals (Charts 1–3), these comparisons bring out the approximately inverse behavior of the seasonal components of some of the items (e.g., Chart 4, Figs. 4, 8, and 11), the timing differences between some other patterns (e.g., Chart 4, Figs. 3 and 5), and the amplitude dominating still other situations (Chart 4, Fig. 1). In comparisons between the figures, which may also be instructive, differences of the amplitude scales ought to be noted.[78]

Of particular interest is the relationship between fresh and canned varieties of the same or similar products, as suggested by Chart 4. The most striking example of an almost perfectly inverse relation found among the materials at our disposal is given in Fig. 8 (Chart 4), where canned apples and applesauce are contrasted with fresh apples. Substantial elements of inverse behavior, however, will also be noted in the comparisons of fresh and canned fish (Chart 4, Fig. 3), fresh and canned tomatoes (Chart 4, Fig. 6), and fresh oranges and concentrated orange juice (Chart 4, Fig. 9).[79] These examples provide strong support for the a priori plausible notion of seasonal substitution between fresh and canned varieties of the same food products or product classes.

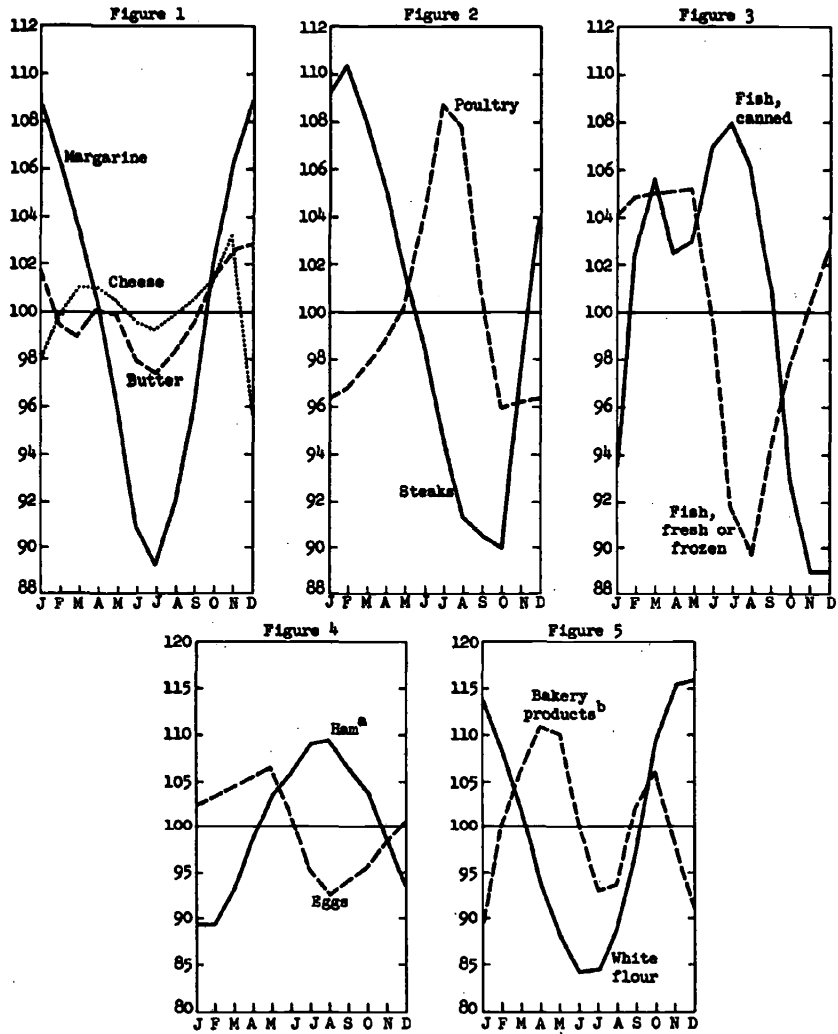
The findings based on the special BLS tabulation are confirmed by independent evidence. Surveys of family food budgets conducted in 1948–49 for the Department of Agriculture provide valuable data on various characteristics of food consumption, including seasonal

[78] Because of the larger seasonal movements in consumption, larger scales had to be used in most of the figures of Chart 3 than were used in Charts 1 and 2.

[79] An inverse relation is also present in the case of grapefruit (Chart 4, Fig. 7), but here it is the contrast between the amplitudes of the two indexes that is the dominant feature of the comparison. (Grapefruit consumption declines to miminal amounts in July–September, and actually this is one of the "seasonal fruits" that are not priced throughout the year by the BLS.) Canned juices show smaller amplitudes and more agreement in the direction of change with the fresh products than do the other canned varieties (cf. Figs. 6 and 9, Chart 4).

CHART 4

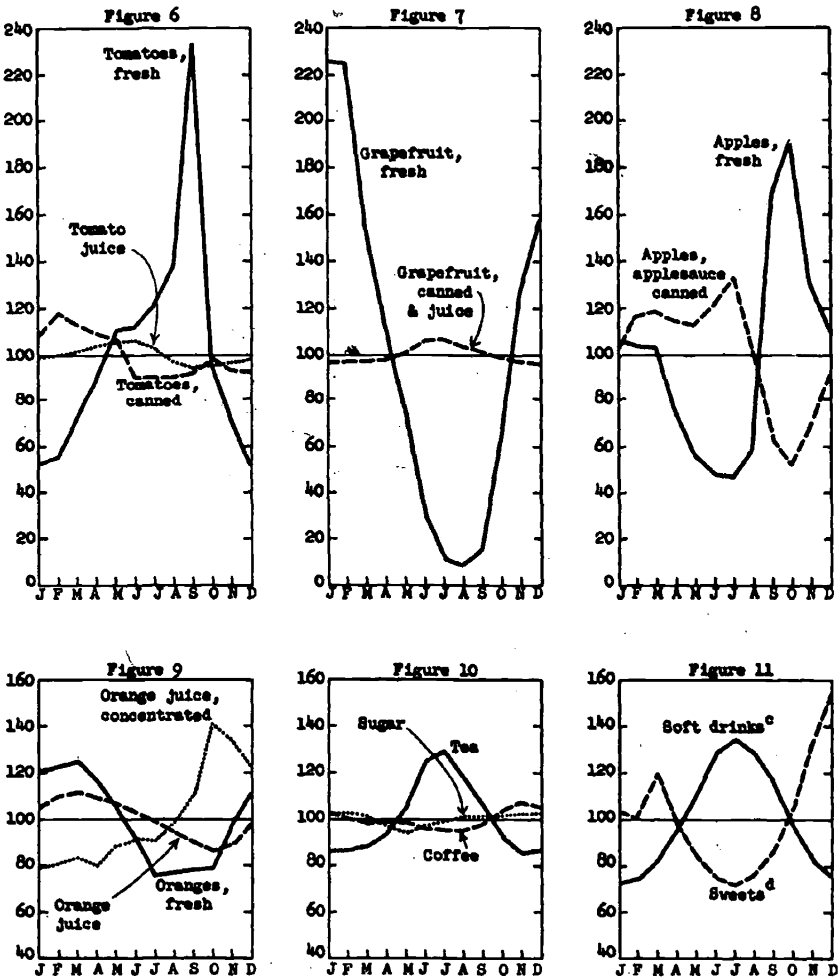
Estimated Seasonal Change in Quantities of Selected Foods Purchased per Month



[a] Whole and sliced; also includes picnics (shoulder).

[b] Cookies, cake, doughnuts, pies, sweet rolls, pastry, and other bakery products.

CHART 4—Concluded



c Cola drinks, other carbonated drinks (excl. ginger ale) and other non-alcoholic beverages (incl. malted milk and powdered fruit drinks).

d Prepared icings, fudge mix, candy, chewing gum, glazed fruit, and other sweets.

variation.[80] Salient features of the four-season indexes derived from these surveys are presented in Table IX. Again, fresh fruits and vegetables are found to be far more sensitive seasonally than any other foods. They reach their seasonal peaks in summer and troughs in winter (except for citrus fruits), which is the opposite of the patterns prevalent among other foods, where summer is the season of the lowest standings. Fresh and processed products have inverse seasonal movements. Also, among the fresh fruits, citrus fruit consumption was seasonally high when the consumption of other fruits was seasonally low, and vice versa. Meat and poultry consumption (low and high in the summer, respectively) interacted in a similar way. For most groups of foods, however, seasonal differences in consumption appear to be relatively small, owing in a large measure to offsetting variations in their individual components.

TABLE IX.—*Measures of Seasonal Variation in Quantities Purchased of Selected Food Items, 1948*

Line	Food item	Seasonal index (year's av.=100)			Change in the seasonal index		
		Highest standing [1] (1)	Lowest standing [1] (2)	Amplitude [2] (3)	Winter-spring (4)	Spring-summer (5)	Summer-fall (6)
1	Milk, cream, ice cream, cheese	105.5 (W)	94.7 (Su)	10.8	−6.7	−4.1	+2.5
2	Bakery products	104.8 (F)	95.9 (Su)	8.9	−4.2	−0.8	+8.9
3	Eggs	106.4 (Sp)	91.5 (Su)	14.9	+3.5	−14.9	+3.9
4	Meats	104.5 (W)	90.6 (Su)	13.9	−4.8	−9.1	+10.0
5	Poultry	112.3 (Su)	96.1 (F)	16.2	+3.7	+12.0	+16.2
6	Fresh fruits	177.1 (Su)	80.9 (W)	96.2	+0.5	+95.7	+84.3
7	Citrus	124.9 (W)	66.0 (F)	58.9	−6.8	−44.0	−8.1
8	Other	245.7 (Su)	51.4 (W)	194.3	+5.4	+188.9	−135.1
9	Canned and frozen fruits	140.7 (W)	59.7 (Su)	81.0	−27.8	−53.2	+0.2
10	Fresh vegetables	122.9 (Su)	80.6 (W)	42.3	+8.4	+33.9	−1.2
11	Canned and frozen vegetables	139.7 (W)	44.1 (Su)	95.6	−29.7	−65.9	+30.5
12	Canned and frozen juices	103.8 (F)	93.2 (Su)	10.6	−4.1	−4.8	+10.6

[1] The seasons corresponding to the figures in these columns are identified in brackets, as follows: W—Winter (Dec.–Mar.); Sp—Spring (Apr.–June); Su—Summer (July–Aug.) and F—Fall (Sept.–Nov.).
[2] Equals the difference between the corresponding figures in cols. 1 and 2.

NOTE: This table refers to "urban housekeeping families of 2 or more persons in the United States."

Source: Based on Table 52 (p. 102) of the Agriculture Information Bulletin No. 132 (1954). See reference in footnote 80.

Regrettably, information on seasonal changes in consumption of products other than foods is exceedingly scanty and inadequate; indeed, what little of it is available to us does not seem to merit presentation. Considerable information on seasonal varieties exists for series on outputs and some for series on inputs and shipments of a variety of products, mostly manufactures. These materials, then, relate to stages preceding consumption or to goods destined for producers rather than consumers. They do throw some light upon the nature of seasonal changes in quantities sold in primary markets by

[80] *Food Consumption of Urban Families in the United States*, by Faith Clark, J. Murray, A. S. Weiss and E. Grossman, Agriculture Information Bulletin No. 132, Home Economics Research Branch, U.S. Department of Agriculture, Washington, D.C., October 1954. This study presents seasonal indexes based on data gathered in the winter, spring, and fall of 1948 and in the spring and summer of 1949. The 1949 data were collected in Birmingham, Ala., and Minneapolis-St. Paul, Minn.; the 1948 data, in the same two cities and also in Buffalo, N.Y., and San Francisco, Calif. In these surveys approximately 4,500 schedules were furnished by households on their weekly food consumption and on certain family characteristics. Careful procedures were followed in combining data for individual food items from the four cities into a single set of weighted seasonal indexes which was described as being fairly representative of U.S. urban consumption. (For the details of the method of constructing these indexes, see the above-cited bulletin, pp. 51–53.)

industrial producers and farmers, although most of the data are for outputs rather than shipments or sales.

A few more general and pronounced characteristics of these output seasonals are brought out in a summary and selective fashion in Table X. There is clearly a considerable degree of similarity between the indexes for several industries. Inspection of the seasonal dia-

TABLE X.—*Highest and Lowest Standings of Seasonal Factors for Selected Components of Federal Reserve Production Indexes*

[Seasonal Index (year's av.=100)]

Industry	1st high [1] (1)	Midyear low [2] (2)	2d high [3] (3)
Primary metals	105	88	101
Fabricated metal products	101	95	104
Nonelectrical machinery	104	95	101
Electrical machinery	102	85	112
Textile mill products	104	85	105
Apparel and allied products	110	85	102
Leather and products	110	88	101
Rubber products	108	82	108
Paper and allied products	104	89	106
Chemical products	102	96	102
Vegetable and animal oils	115	76	121
	1st low	Midyear high	2d low
Food manufactures	[4] 91	[5] 117	[6] 97
Beverages	[7] 80	[8] 119	[6] 84

[Seasonal Index (year's av.=100)]

Consumer durables	1st high [9] (1)	Midyear low [10] (2)	2d high [11] (3)
Autos [12]	115	56	121
Household furniture	100	95	106
Floor coverings	109	76	108
Refrigeration appliances	127	73	96
Laundry appliances	114	71	113
Radio sets [12]	105	56	139
Television sets [12]	106	59	135
Miscellaneous home and personal goods	100	93	108
	1st low	Midyear high	2d low
Auto parts and tires	[4] 95	[5] 109	[6] 97

[1] Standings in March (6), February (3), January (1), and April (1).
[2] All July standings except for one August (nonelectrical machinery).
[3] Standings in October (9), November (1), and December (1).
[4] March.
[5] September.
[6] December.
[7] January.
[8] July.
[9] Standings in February (4), March (3), and April (1).
[10] Standings in July (6), August (1), and September (1).
[11] Standings in October (1), September (2), and November (2).
[12] 1957 indexes used (1955 and 1956 indexes slightly different).

NOTE.—The indexes are those for 1955-57 or 1956-57, except as indicated in note ([12]).

Source: Division of Research and Statistics, Board of Governors of the Federal Reserve System Seasonal Adjustment Factors, 1947 to 1957, Federal Reserve Production Indexes (May 1959 mimeo.).

grams for the major components of the Federal Reserve production indexes shows that common to most of them is a broad "double-peak" pattern of fluctuation. A peak or high standing of the seasonal in the first quarter of the year is followed by a descent to a summer vacation trough, mostly in July, which is often conspicuously low. Then there is a rise to a second peak in the last quarter. Otherwise the patterns vary greatly; for example in some the first peak is higher, in others the second. Outputs of major consumer durables show particularly large movements of this type, except for automobiles, where the nadir occurs at the model-changeover time, now early in the fall. Products processed from agricultural raw materials show less but still relatively high seasonal sensitivity. Some of them, such as vegetable and animal oils, conform to the double-peak model. But food manufactures and beverages have entirely different patterns, with single peaks in September and June, respectively.

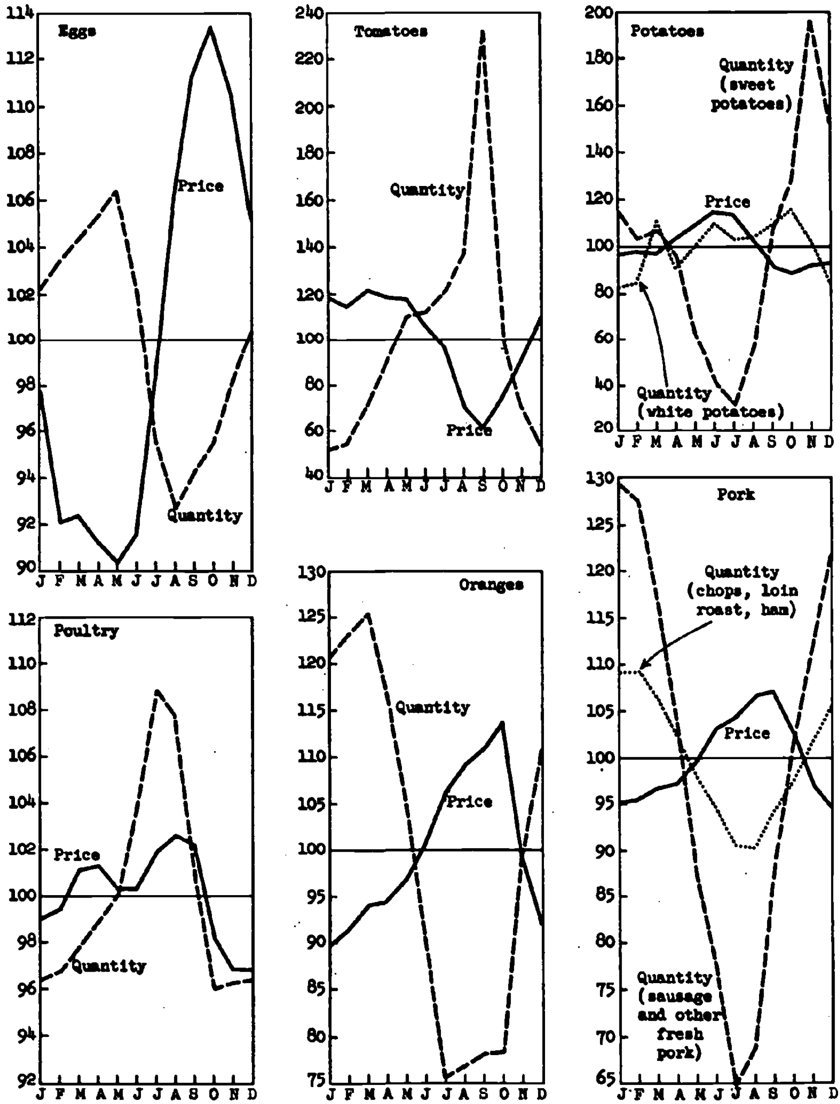
5. PRICE-QUANTITY INTERACTIONS OBSERVED IN SEASONAL PATTERNS

The seasonal indexes that we were able to collect for this study offer few possibilities of even roughly matching the data on prices and quantities by product and transactor characteristics. A few examples for food products are shown in Chart 5. The price data are seasonal factors for selected CPI components; the indexes for quantities consumed came from the special BLS tabulation (see Section 4 above).

As illustrated by Chart 5, the evidence for food products confirms what would be expected on theoretical grounds, viz, that seasonal movements in the prices and quantities of many goods are inversely correlated. The negative relationships are very pronounced indeed for such highly seasonal commodities as eggs, tomatoes, and oranges. The evidence for some food groups—dairy products, meats, and fish—is somewhat mixed, but here too elements of negative association seem to prevail and are sometimes very strong (as in the case of pork shown in the chart). In some instances, however, relations that are on the whole positive rather than negative are found. The best example for this that we could establish is given in the diagram for poultry in Chart 5. This also is in accord with theoretical considerations. As noted before, where seasonal changes in prices and quantities are due to shifts in demand rather than in supply curves one would expect the seasonal price-quantity relationship to be positive, not negative.

CHART 5

Seasonal Movements in Quantities Purchased and Prices for Selected Food Products



6. COMMODITIES NOT PRICED OR NOT AVAILABLE IN CERTAIN SEASONS

Some items of extreme seasonal sensitivity are not directly priced throughout the year in the process of compiling the price index series. The treatment of such "part-year" commodities in the major U.S. price indexes has been broadly discussed in Part II, Section 2; in what follows these items and their principal characteristics will be identified for each of the indexes under review.

TABLE XI.—*Items With Pricing Seasons of Less Than a Year in the U.S. Consumer Price Index*

Line	Group	Commodity	Specification no.	Pricing season
1	Food, fresh fruits	Grapefruit	F-423	November-May.
2	do	Peaches	F-425	July-September.
3	do	Strawberries	F-426	April, May, and June.
4	do	Grapes	F-427	July-November.
5	do	Watermelons	F-428	June, July, and August.
6	Apparel, Women's and girls'	Coat, fur	A-407	September-January.
7	do	Coat, without fur trim	A-410	Do.
8	do	do	A-415	Do.
9	do	Dress, all new wool	A-490	Do.
10	do	Coat, all new wool, girls'	A-600	Do.
11	do	Skirt, all new wool, girls'	A-620	Do.
12	do	Sweater, Orlon, girls'	A-632	Do.
13	do	Suit, all new wool	A-431.1	September-April.
14	do	Suit, rayon acetate	A-441	Do.
15	do	Dress, cotton, street	A-495	March-July.
16	do	Coat, sport, light	A-420	February-April.
17	Apparel, Men's and boys'	Sweater, all new wool	A-141	September-January.
18	do	Jacket, Gabardine rayon acetate.	A-150 ser.	Do.
19	do	Jacket, rayon, boys'	A-340 ser.	Do.
20	do	Topcoat, all new wool	A-101	September-March.[1]
21	do	Suit, all new wool, boys'	A-310	Do.
22	do	Shirt, sport, long-sleeve, men's.	A-213.1	Do.
23	do	Shirt, sport, long-sleeve, boys'.	A-371	September-February.[1]
24	do	Suit, tropical, worsted	A-118	March-July.
25	do	Suit, rayon tropical	A-120	Do.
26	do	Shirt, sport, short-sleeve, men's.	A-212	April-August.[1]
27	do	Shirt, sport, short-sleeve, boys'.	A-370A	March-August.[1]

[1] Approximately.

Source: Bureau of Labor Statistics, U.S. Department of Labor.

a. *Consumer Price Index.*—Until 1953 the group of "part-year" commodities in this index consisted only of certain apparel items; since that time, a few food items—all fresh fruits—have been added. The list now includes five fruits, eleven items of women's and girls' apparel, and eleven items of men's and boys' apparel. These commodities and their respective pricing seasons are identified in Table XI.

The present procedure is for the seasonally disappearing apparel items to be estimated during their off-season periods by the movement of the year-round apparel products.[81] The method used for the food items is somewhat different. Here those commodities that cannot be priced directly in a given month have their price movements estimated by the change in price of total fresh fruits, including not only the year-round items but also those "part-year" fruits for which direct prices are available in the months concerned. The food method utilizes more information than the apparel method but it thus strengthens the influence of the highly seasonal "part-year" fruit items which are extremely variable and at times volatile. As a result, very large price relatives for fresh fruits are reflected in the index at the beginning of the season for such commodities as peaches, grapes, and watermelons, i.e., in the months of June and July when these fruits are still expensive. For this reason, we are informed, the BLS is considering the advisability of applying the procedure now used for apparel to the seasonal fruits as well.

[81] Before 1953 these prices were assumed to undergo no change off-season.

Large price declines usually occur between the first and second month of pricing a seasonal item such as any of the fruits listed in lines 1–5 of Table XI. The other fresh fruits do not show such declines at these times (see the accompanying tabulation).

Retail Price Relatives, Chicago

Year	Strawberries (April-May)		Peaches (July-August)		Watermelons (June-July)		Grapes (July-August)	
	Actual	Year-round fresh fruits	Actual	Year-round fresh fruits	Actual	Year-round fresh fruits	Actual	Year-round fresh fruits
1956	68.3	108.1	92.9	91.3	82.1	106.0	65.7	91.3
1957	71.4	94.2	79.7	86.4	86.9	109.7	60.2	86.4
1958	78.9	100.5	89.4	81.9	71.1	105.3	75.4	81.9
1959	70.3	102.2	79.9	100.6	69.5	97.5	67.8	100.6

Taken at their face value, these comparisons would seem to suggest that the errors involved in the imputation procedure as applied to the above items are very substantial. However, it is important to note that this is surely an extreme test of the possible imputation errors, since it is restricted in each case to a single month-to-month interval which, in the present context, has very special characteristics.[82]

Chart 6 presents monthly retail price relatives (Chicago, 1955–58) for all "part-year" items in the fresh fruits group and about half of those in the apparel group.[83] The chart shows very large up and down movements of fruit items during their pricing seasons and suggests that these movements often influence strongly the behavior of the total fresh fruits index. The apparel items, on the other hand, are very stable, their price relatives being frequently equal to 100, or approximately so, for several months.[84] (Prices of women's and girls' apparel are appreciably less stable than those of men's and boys' apparel). It should make little difference whether the off-season prices of these items are varied with the apparel group index or are held constant at their end-of-season levels.

A limited objective that the index maker may wish to pursue is to avoid sudden "breaks" in the series at the time a commodity reappears after its off-season period. This can be achieved retroactively through periodic revisions in which estimates for the seasonally disappearing items that are based on interpolation between the initial and the terminal dates of their respective off-season periods would be substituted for the original estimates based on extrapolation from the former dates. Another practical consideration is that the imputation procedure can be expected to present less difficulty when it is applied

[82] Another qualification, believed to be minor, is that the tabulation in the text lists the relatives for the year-round fruits only, whereas in the actual BLS procedure the relative used to estimate items during the off-season is based on a combination of year-round items and any of the "part-year" goods priced in the current month. (If one assumes that this procedure is extended to the first two months of pricing a seasonal item, then the price of the latter should, strictly speaking, be omitted from the estimating relative for these months.) We are indebted to Mr. Sidney A. Jaffe of the BLS for both the figures used in the tabulation above and the critical remarks on the significance of these comparisons.

[83] It should be noted that the price relatives in the first month of the pricing season are composed differently for fruits and for apparel. Those for fruits represent the change from the previous month's implicit price, which is the estimated price obtained by continuous application over the off-season period of the price relatives for all priced fresh fruits. Those for apparel represent the change from the end of the previous pricing season.

[84] The items included in Chart 8 are in this respect representative of those that have been omitted.

CHART 6

Monthly Retail Price Relatives for Seasonal Fruits and Apparel, Chicago, 1955-58

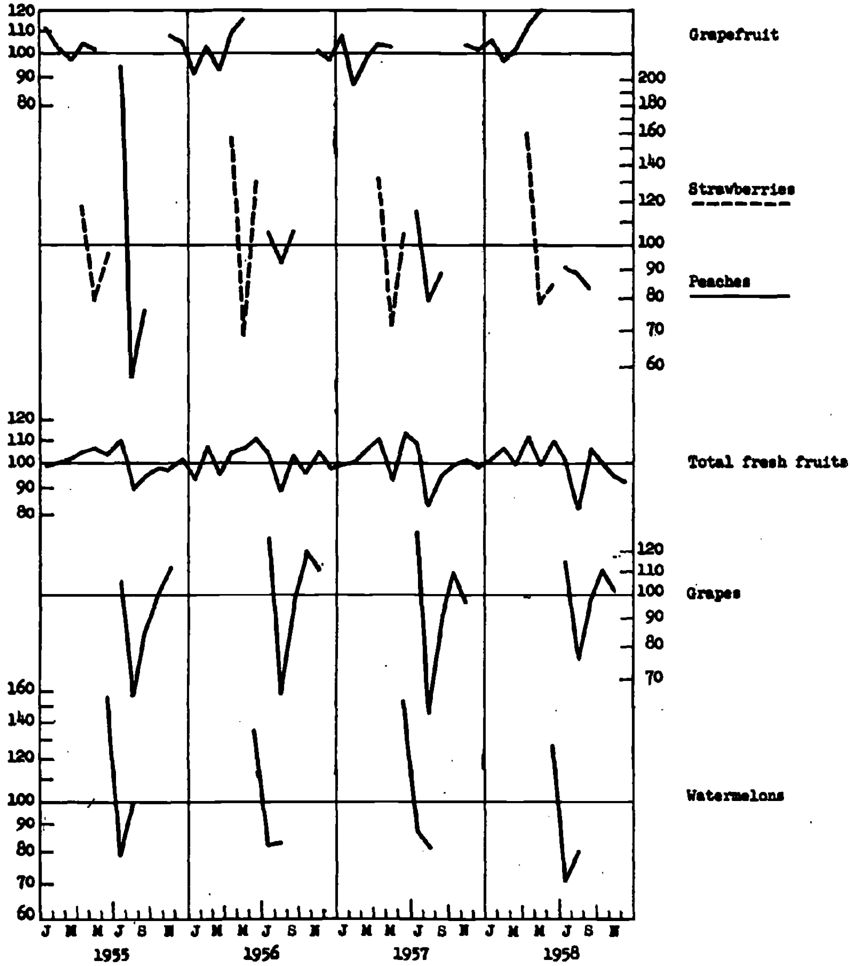
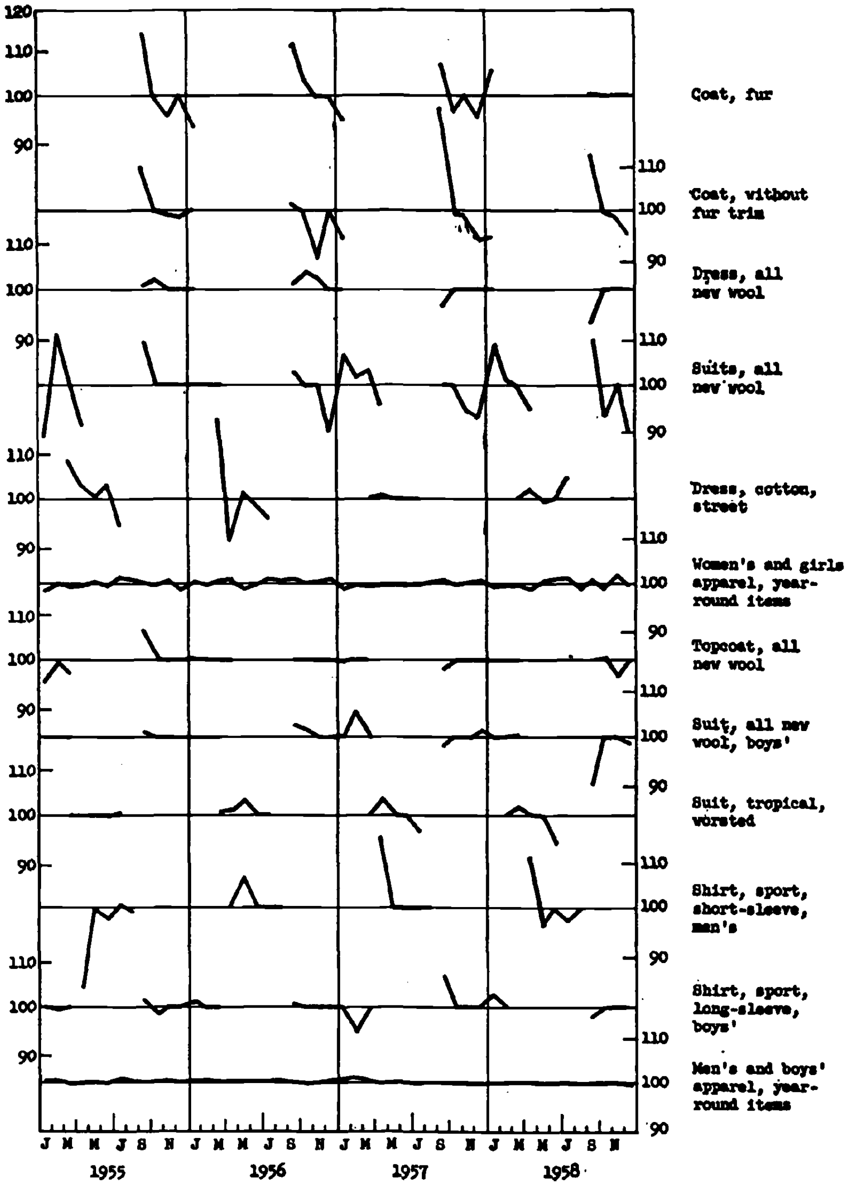


CHART 6—Concluded.



to seasonally adjusted price series than when unadjusted data are used. This is because the month-to-month changes are smaller in the former series and because elimination of their different seasonal components is likely to reduce the dissimilarity between the series used in the imputation procedures.[85]

b. *Wholesale Price Index.*—This index includes twenty-four "part-year" items evenly divided between a group of farm products (all fresh foods) and a group of apparel and textiles. These commodities and their pricing seasons are identified in Table XII.

TABLE XII.—*Items with Pricing Seasons of Less Than a Year in the U.S. Wholesale Price Index*

Line	Group and subgroup	Commodity	Code	Pricing season [1]
1	Farm products; Fresh fruits	Apples, Delicious	01-11-01	October-May.
2	Farm products; Fresh fruits	Apples, Winesap	01-11-02	March-August.
3	Farm products; Fresh fruits	Grapefruit, Florida	01-11-21	October-June.
4	Farm products; Fresh fruits	Oranges, Florida	01-11-26	October-July.
5	Farm products; Fresh fruits	Grapes	01-11-31	July-March.
6	Farm products; Fresh fruits	Peaches	01-11-36	July-September.
7	Farm products; Fresh fruits	Pears	01-11-41	July-May.
8	Farm products; Fresh fruits	Strawberries	01-11-51	Apr.-Aug., Nov.-Jan.
9	Farm products; Fresh and dried vegetables.	Cantaloupes	01-13-21	April-October.
10	Farm products; Live poultry	Turkeys, hens	01-32-80	June-January.
11	Farm products; Live poultry	Turkeys, toms	01-32-85	June-January.
12	Farm products; Oilseeds	Cottonseed	01-73-21	July-March.
13	Apparel; Women's and misses'.	Women's coat, trimmed	03-51-12	July-October.
14	Apparel; Women's and misses'.	Women's coat, untrimmed	03-51-14	May-December.
15	Apparel; Women's and misses'.	Women's skirt	03-51-62	January-May.
16	Apparel; Women's and misses'.	Women's skirt	03-51-66	May-December.
17	Apparel; Infants' and child	Girls' coat	03-54-12	May-December.
18	Apparel; Men's and boys'	Men's suit	03-52-06	September-April.
19	Apparel; Men's and boys'	Men's suit	03-52-07	October-May.
20	Apparel; Men's and boys'	Men's topcoat	03-52-12	July-October.
21	Apparel; Men's and boys'	Men's sport shirt	03-52-36	January-April.
22	Apparel; Men's and boys'	Boys' cotton broadcloth shirt	03-52-41	December-April.
23	Apparel; Knit underwear	Boys' polo shirt	03-56-15	November-February.
24	Textile products; Broad woven goods.	Tropical blend fabrics	03-33-32	June-March.

[1] Pricing seasons for food items are somewhat flexible, depending upon supply. Pricing seasons for apparel items are approximate, varying slightly for individual firms.

SOURCE: Bureau of Labor Statistics, U.S. Department of Labor.

Until April 1959, prices of these items during off-season months were held constant. After that date, the practice regarding farm products was changed; their off-season prices are now imputed to the movement of the product class in which they fall. The constant off-season price method, however, is still used in the WPI for the apparel items.

c. *Prices Received by Farmers.*—Among farm products priced for this index are some that have short marketing seasons. For these commodities—the tobaccos, cottonseed, and seven fruit and vegetable crops—current prices are not available on a year-round basis.

In the case of tobacco, average prices for the most recent season are used for those types not currently sold. These are included along with the actual current prices of the actively marketed types in the average price of tobacco as a whole. The weights used in the computation of this U.S. average price are in all months the annual produc-

[85] The observations made in this paragraph of the text apply to the imputation method generally. They are thus equally pertinent to the problem of seasonal disappearances in the WPI (to be discussed presently) as they are to the same problem in the CPI.

tion estimates for the various tobacco types. As explained in a statement received from the Agricultural Marketing Service, use of the average price of current sales as the index price would result in drastic month-to-month changes due to shifts in the types being sold during different seasons.

In the case of cottonseed and the fruit and vegetable crops with short marketing periods (varying from 4 to 11 months), the price of the last month of the season is used in the index until the next crop starts to be marketed. The AMS statement notes that the use of the season average in the off-months of marketing (as in the case of tobacco) would here result in rather sharp shifts in price from the last price of the season toward that average and then again from the latter toward the first price of the new season. The practice of using the price of last month of marketing apparently causes fewer shifts and is thus considered preferable.

7. POSSIBLE IMPROVEMENTS AND FURTHER RESEARCH

It is clear that pronounced seasonal movements are characteristic of many price series and that they should not be ignored.[86] As a minimum, the series should be prepared and published in the seasonally adjusted as well as unadjusted form. True, given the present systems of fixed annual weights, the aggregate price indexes at our disposal are not truly "unadjusted" and mere application to such group or overall indexes of some standard statistical "deseasonalization" methods cannot assure us of the precise meaning and quality of the resulting "seasonally adjusted" series. But by adjusting the individual series and combining them with annual weights, aggregative indexes can be produced that may in practice be quite satisfactory as measures of the nonseasonal price change. There is obvious need for such measures and their regular calculation would, in this writer's view, be very desirable.

Beyond this, any possible improvement on a larger scale would involve the use of seasonal weights and be far more difficult and costly to achieve. But we do not face an "all or nothing" alternative in this area. The advance can be partial and yet significant, and the studies needed for a detailed decision of what can and should be done would be of great interest in themselves.

We need to know more about how stable the seasonal patterns of change in prices and quantities are over time. It is possible and rather likely that sufficiently stable and pronounced patterns exist for some part of the commodity universe but not for the rest of it. To identify these two parts would then be an essential prerequisite for a practical program of constructing a seasonal price index. For the portion of the index with large and stable seasonalities, a fixed-base, seasonal-weight formula would be appropriate. For the portion with small or variable seasonalities, annual weights would probably have to be retained, since chain indexes with seasonal weights are not likely to offer a practical solution. Periodic corrections of the results, perhaps with the aid of independently determined annual averages, are compatible with the seasonal procedures suggested and would pre-

86 The problem of seasonally vanishing goods, in particular, cannot be avoided. Having given it much attention before, we need not return to it in these concluding remarks, except to say that the treatment of these commodities must be a compromise but as such should be made as logical and consistent as possible.

sumably be needed. Indeed, a separation between monthly estimates and annual series may prove necessary if the requirements on a monthly series could not be met.

The most ambitious undertaking that might be considered in this area is an attempt to identify a basic set of seasonal market baskets of equivalent utility contents. To prepare the way for it, all available information bearing on seasonalities in quantities and prices would need to be brought together and appropriately systematized by groups with different degrees of substitutability. Existing studies of demand elasticities, etc., should be utilized. At the least, this work would indicate the dimension of gaps in our present knowledge that future effort should be directed to close. At the most, probably, the study would yield encouraging indications that the project can be accomplished within a reasonable period of time rather than being only of remote feasibility.